On the Mapping between Logical and Physical Topologies

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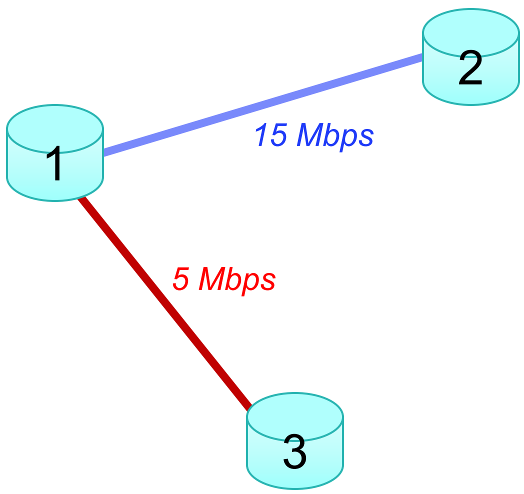
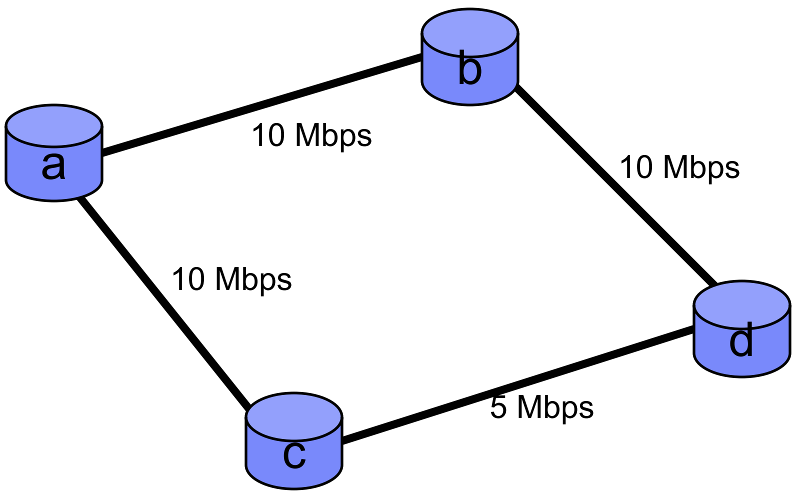
Graph Mapping Problem



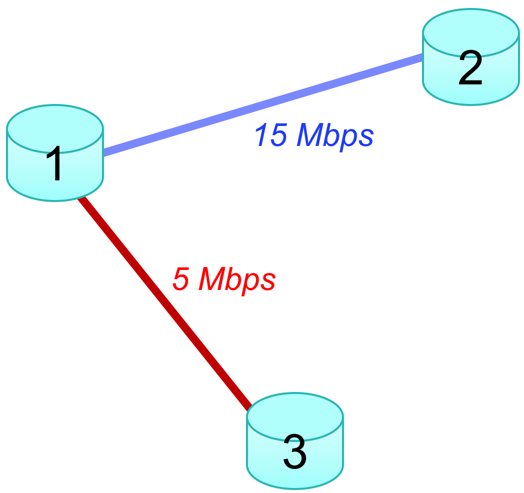
Demand
(Logical network)



Constraints
(Physical network)

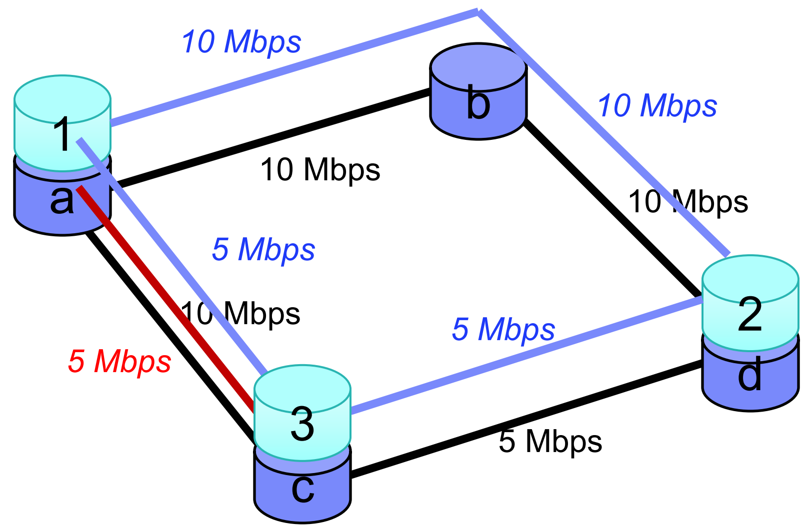
Question: How to map logical network to physical network such that the demand is satisfied?

Graph Mapping Problem



Demand

(Logical network)



Constraints

(Physical network)

Question: How to map logical network to physical network such that the demand is satisfied?

Motivations

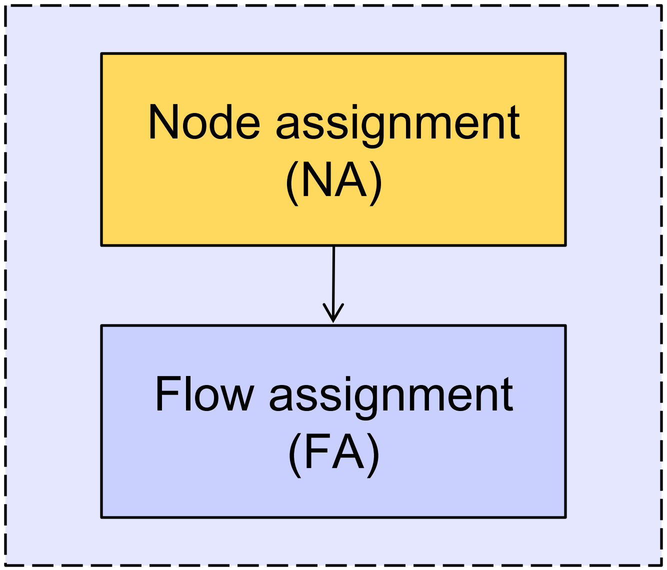
- Cloud computing
- Social networking
- Smarter networking
- Related work
  - Task-processor assignment
  - Wavelength assignment in optical networks (WDM)
  - Multi-commodity flow optimization

Problem Statement

- Given
  - A logical graph $G_L = (V_L, X)$
  - A physical graph $G_P = (V_P, C)$
- Determine
  - Node assignment: place 1 logical node at 1 physical node.
  - Flow assignment: find a (multi-path and multi-hop) routing that satisfies the logical requirements and physical capacities.

The two-step approach

- To solve the problem, we take a 2-step approach



NA : Find a feasible node assignment

Feasible NA: under the NA, one can find at least one feasible routing

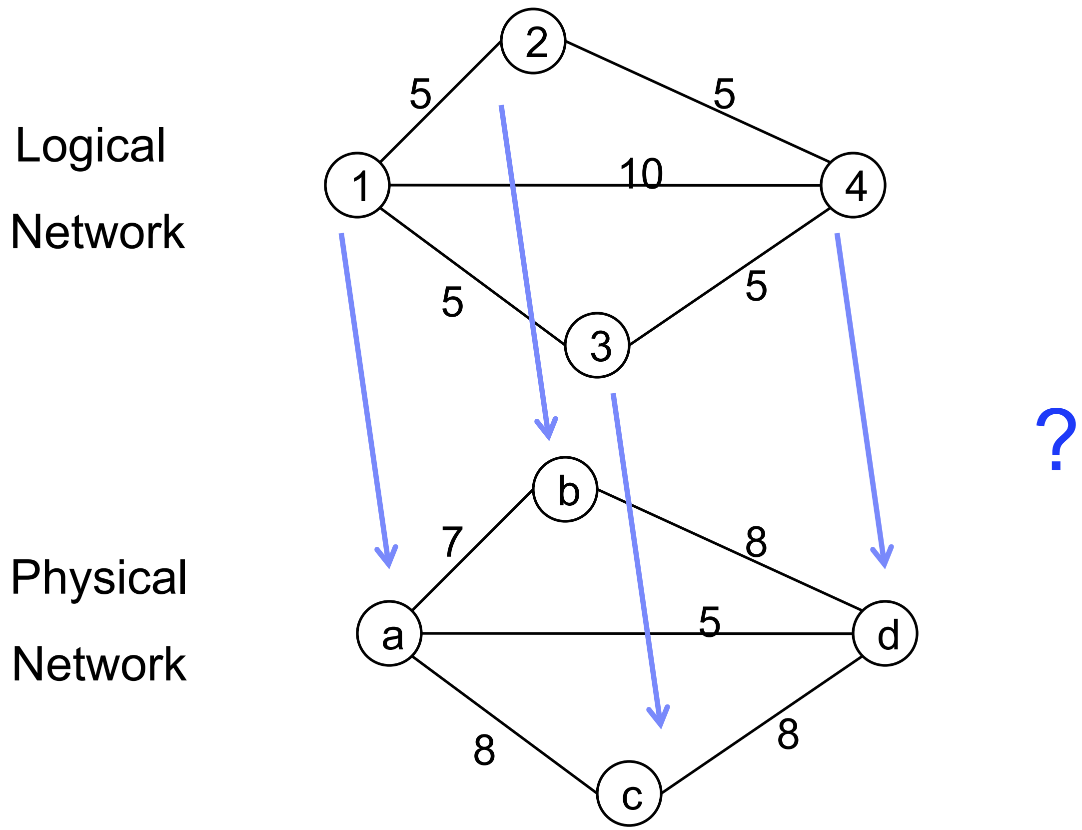
FA : Find a feasible routing

Feasible routing: satisfies the physical capacities while well meets the logical requirements

When NA is given, FA is the well known Multi-Commodity Flow problem

Our contribution: conditions of NA feasibility and how to find out the feasible NA

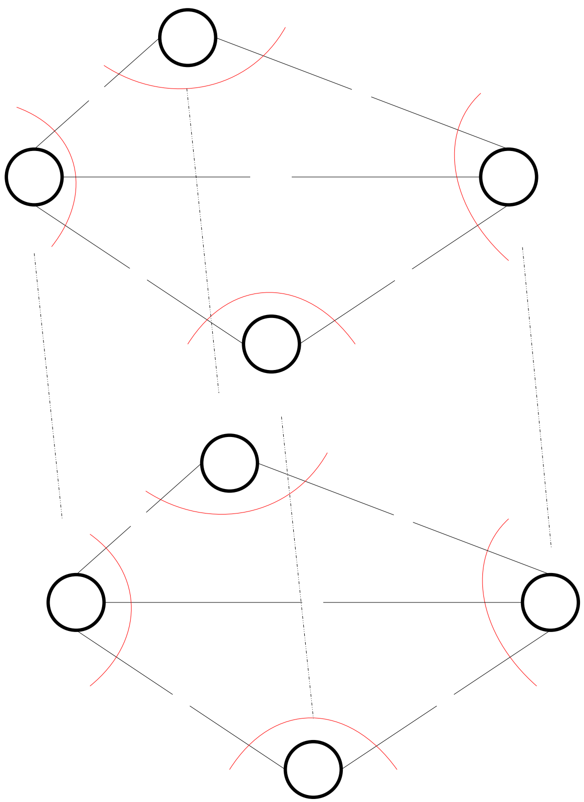
Pop quiz



No, because the demand cannot be met…

Logical Network

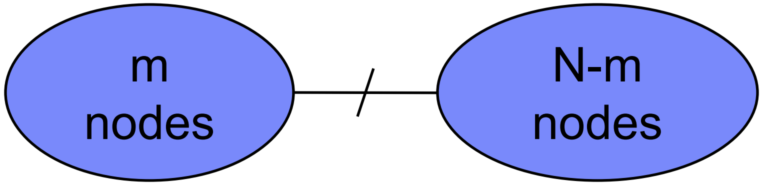
Physical Network



All 1-cut checks are passed

Feasibility testing - the m-cut checks

- Definitions
  - Cut = a set of edges that partitions a graph into two sides
  - m-Cut: a cut that partitions the graph into a set of m nodes and the other set of N-m nodes
  - Cut capacity: sum of the weight of edges in the cut



- The m-cut feasibility check
  - A given NA and a random m-cut
  - if the PHY m-cut capacity $\geq$ the LOG m-cut capacity, the check is passed

Good news and bad news

- Feasibility condition for NA

  Consider m-cut feasibility checks for m = 1, 2, …, N/2. If all the checks are satisfied, the particular node assignment is feasible; vice-versa, if a particular node assignment is feasible, it must pass all the m-cut checks.

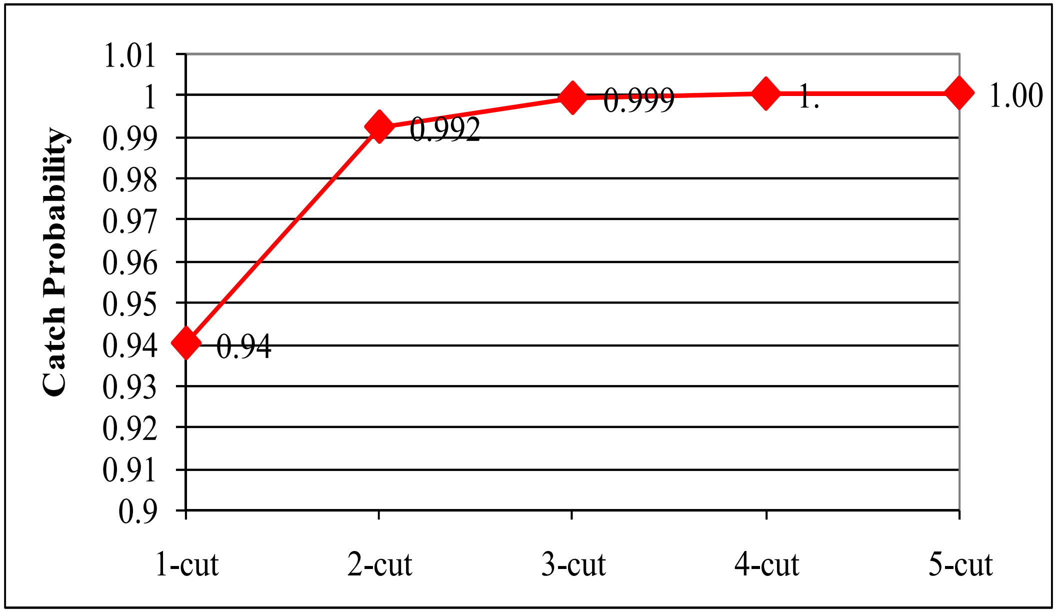
- This condition is shown to be necessary and sufficient using max-flow theory

- There are $C_N^m = \binom{N}{m}$ m-cuts!

NA algorithm 1
- Random Assignment Plus Check (RAPC)

- The all-cut RAPC algorithm
  1) Pick up a random node assignment
  2) Check all m = 1,2,…,N/2 cut checks
- Complexity of RAPC
  – If two network can be mapped by some NA
    - The RAPC goes over k-1 unfeasible NA's
    - Reach a feasible NA at k-th trial
    - Complexity : $O(2^N)$
  – If two networks cannot be mapped via any NA
    - The RAPC goes over all possible N! NA's
    - Complexity : $O(N!)$

Catch probability P_m



Random generated PHY and LOG of size *N* and *M* in the range of 2 – 10

Uniformly distributed edge weights

NA algorithm 2
- Simplified RAPC (1-cut RAPC)

- The 1-cut RAPC algorithm
  1) Pick up a random node assignment
  2) Check the all 1-cut check
  3) If all the1-cut checks are passed, the NA is asserted to be feasible
- Complexity of 1-cut RAPC
  - If two network can be mapped by some NA
    - Complexity : $O(N)$
  - If two networks cannot be mapped via any NA
    - The RAPC goes over all possible N! NA's
    - Complexity : $O(N!)$

NA algorithm 3
- Greedy 1-cut mapping (1-cut MA)

- Instead of random assignment, we assign nodes in a greedy manner
- Greedy 1-cut algorithm
  1) Sort the logical nodes in descending order of 1-cut logical capacity $\beta_1(1) \geq \beta_1(2) \geq \cdots \geq \beta_1(m) \geq \cdots \geq \beta_1(M)$
  2) Sort the physical nodes in descending order of 1-cut capacity $\lambda_1(1) \geq \lambda_1(2) \geq \cdots \geq \lambda_1(n) \geq \cdots \geq \lambda_1(N)$
  3) Starting from k = 1, map the k-th PHY node to the k-th LOG node, if $\lambda_1(k) \geq \beta_1(k)$; if $\lambda_1(k) < \beta_1(k)$, stop the algorithm
- If a NA is formed by 1-cut MA, it must be feasible
- Complexity of 1-cut MA = $O(N)$

Simulation results - Complexity of the algorithms

- A specific case for illustration
- When M=N=4

		Complexity	Error Pr
$(N, M) = (4, 4)$	All-cut RAPC	566	0
	1-cut RAPC	523	5.3%
	1-cut MA	4	5.3%

1-cut MA algorithm is dramatically faster than RAPC algorithms

The high complexity of RAPC is due to the NP-hard nature of the problem

Error-tolerant scenarios → choose 1-cut MA

Error-sensitive scenarios → choose all-cut RAPC

Performance evaluation - Error probability

- How accurate are the algorithms?
- Erroneous decision: an NA is unfeasible, but the algorithm identifies it as feasible
- Error prob. = prob. of making erroneous decisions

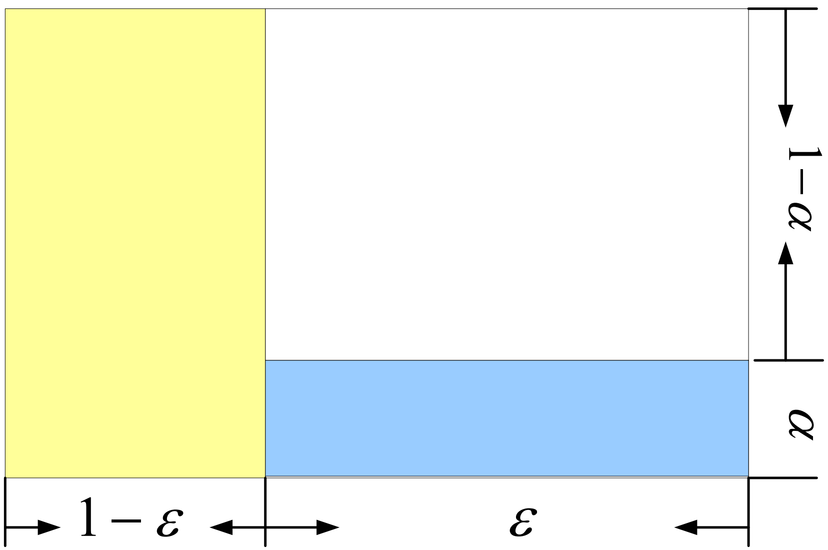
All-cut RAPC: $P_e = 0$

1-cut RAPC: $P_e = (1-\alpha\varepsilon)(1-P_1)$

1-cut MA: $P_e = (1-\alpha\varepsilon)(1-P_1)$

P_1 = prob {the 1-cut check is failed | the NA is unfeasible}

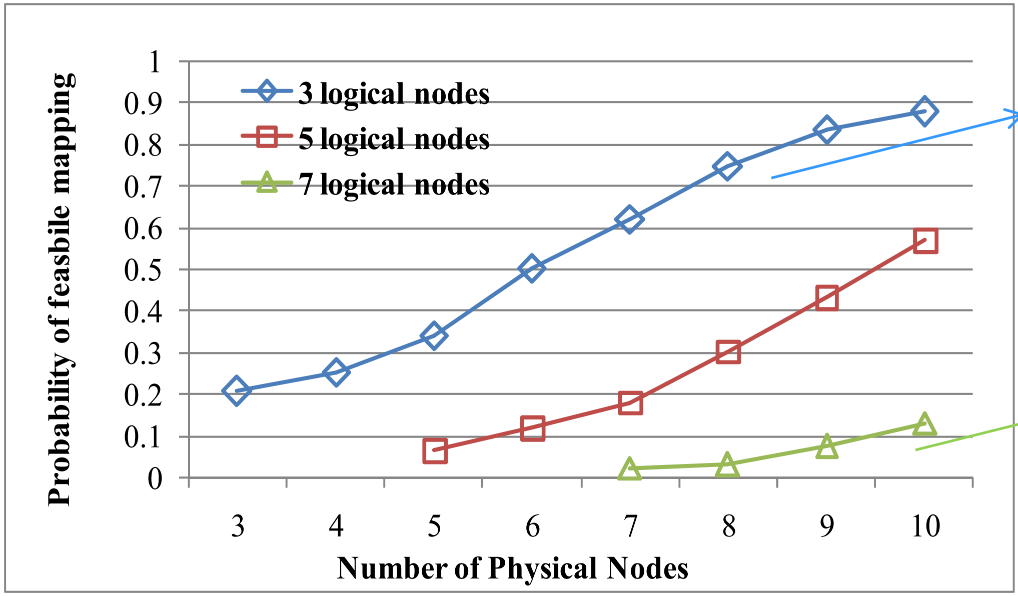
$1-P_1$ = the prob that the 1-cut cannot catch an unfeasible NA



How effective is the 1-cut? = How big is P_1?

Simulation results - Probability of feasible mapping

- For two randomly generated network, how often can they be mapped (there is at least one NA feasible)?



High redundancy

1 extra node provides large increase

Low redundancy

1 extra node provides small increase

The more redundancy → the higher prob of feasible mapping

Gain by redundancy is inversely proportional to the size of logical network

Summary

- This paper
  - Presented a novel set of feasibility checks for node assignments based on graph cuts
  - Showed the conditions to be necessary and sufficient.
  - Proposed a simple and fast algorithm for node assignment based on 1-cut
- Possible next steps
  - Dynamic assignment problem
  - Node assignment with capacity consideration
  - Multiple LOG mapping
  - Other interesting issues

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