State Dependent Attempt Rate Modeling of Single Cell IEEE 802.11 WLANs with Homogeneous Nodes and Poisson Arrivals

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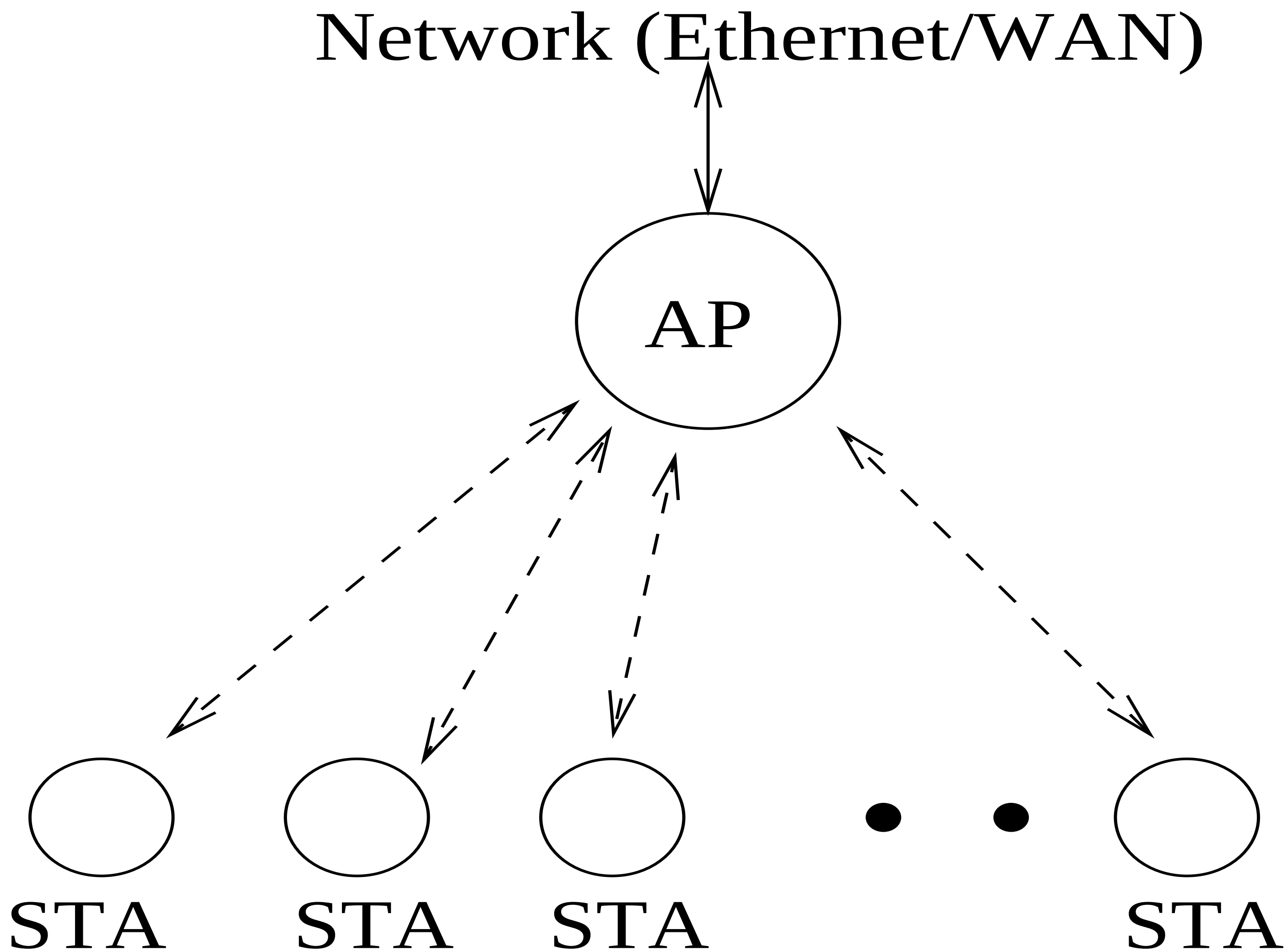
Outline

- Network Model and Assumptions
- A Coupled Queue Model
- Reduction of the State Space
- Derivation of Performance Measures
- Numerical and Simulation Results
- Conclusion

Network Model and Assumptions

An Example Single Cell Network



The Network Model

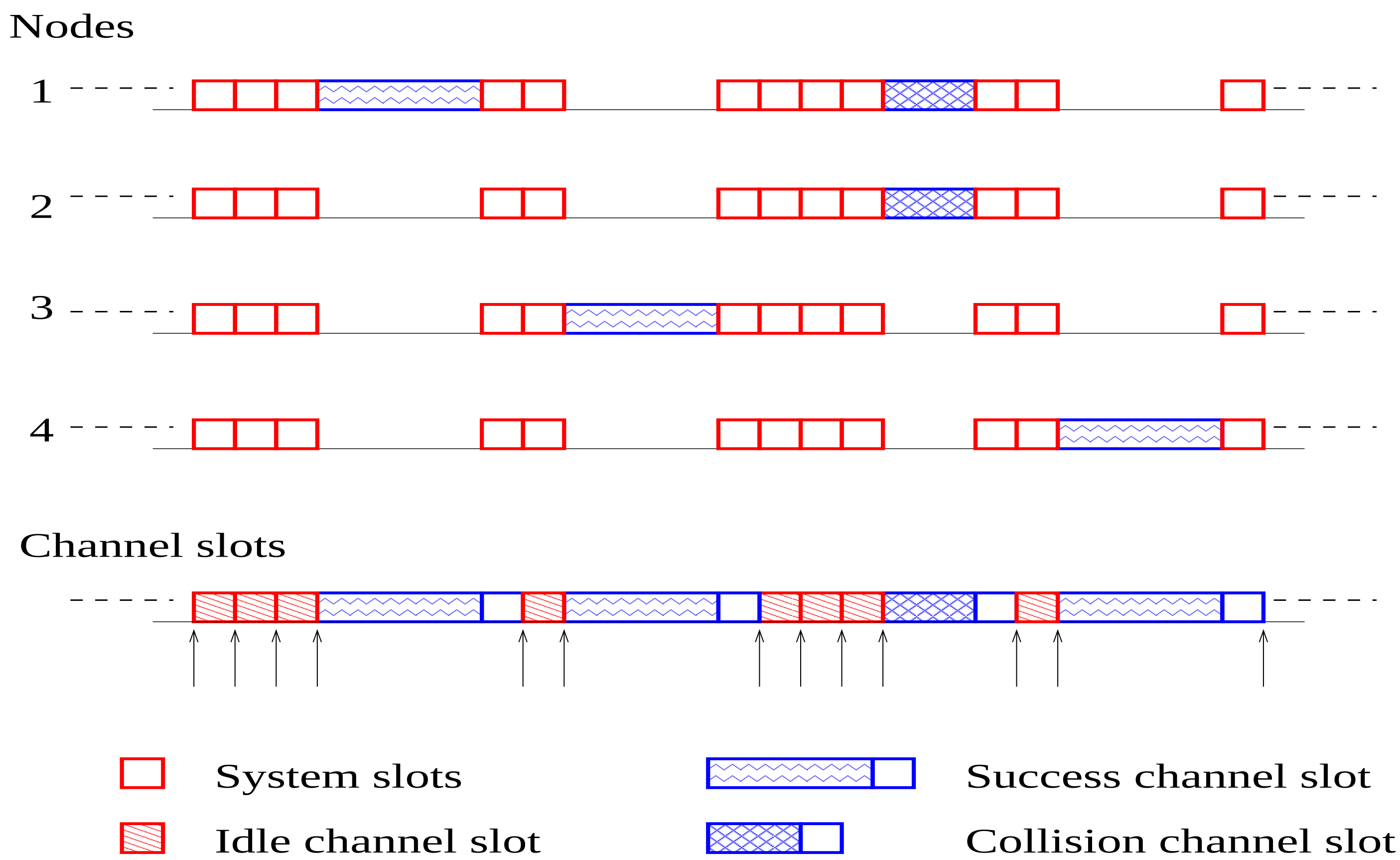
- Error-free Wireless Channel
  - single transmissions are always successful
- No Hidden Nodes
  - packet loss only due to simultaneous transmisssions
- No Capture
  - simultaneous transmissions $\Rightarrow$ all are lost
  - at most one successful transmission at any point of time

Assumptions

- M Homogeneous Nodes
  - identical protocol parameters
- Packet Arrival Process
  - independent Poisson processes with rate λ packets/sec
  - fixed size packets
- Infinite MAC buffer
  - packets are never dropped due to buffer overflow

A Coupled Queue Model

Channel Slots

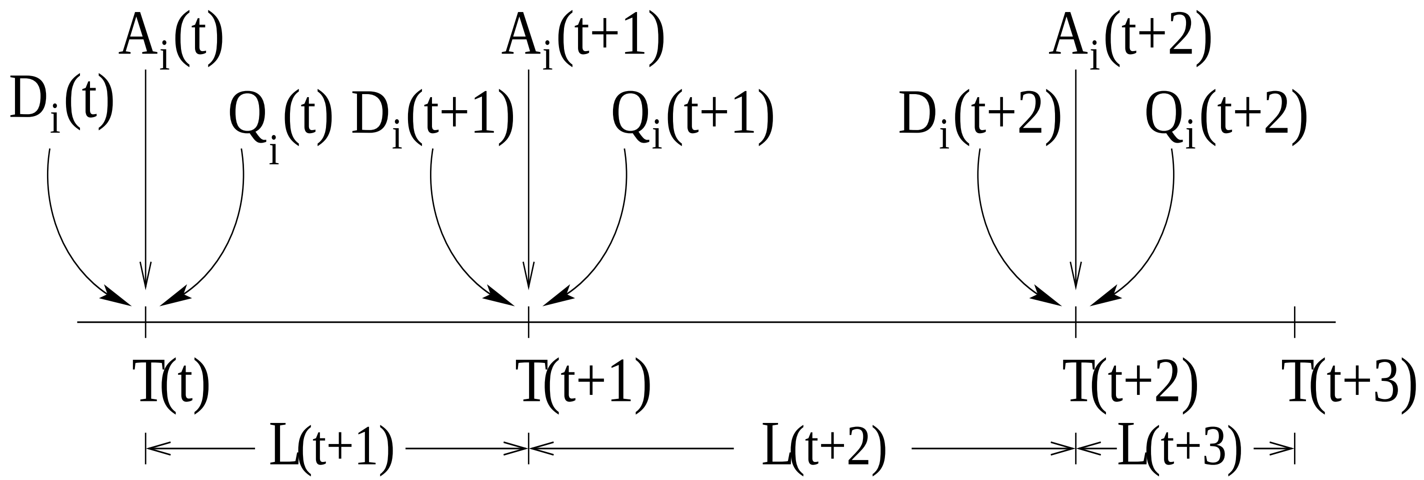


- Length of Channel Slots: $L_{idle}, L_{succ}, L_{coll}$

Attempt Model

- Saturated Case [Bianchi]
  - constant attempt probability β
  - not accurate when nodes are not saturated [Garetto et al.]
- The SDAR Approximation [Kuriakose et al.]
  - when n nodes are non-empty, attempt with probability β_n
  - β_n : long-term attempt rate of n saturated nodes

Evolution of the System



- $T(t), t = 0, 1, 2, \cdots, T(0) = 0$: channel slot boundaries
- $L(t)$: duration of the t^{th} channel slot
- $Q_i(t)$: # packets in queue at time t; $Q_i(t) \in \{0, 1, 2, \cdots\}$
- $A_i(t)$: # arrivals in the t^{th} channel slot; $A_i(t) \in \{0, 1, 2, \cdots\}$

- $D_i(t)$: # departures in the t^{th} channel slot; $D_i(t) \in \{0, 1\}$
- $N(t)$: # non-empty nodes at time t

$$N(t) = \sum_{i=1}^{M} \mathbf{1}_{\{Q_i(t)>0\}}$$

- The $\{Q_i(t)\}$ processes evolve as:

$$\boxed{Q_i(t+1) = Q_i(t) - D_i(t+1) + A_i(t+1)}$$

Convention : $Q_i(t) = 0 \Rightarrow D_i(t+1) = 0$

- $d(j)$: probability that j pkts arrive in a slot given idle

$$d(j) = e^{\lambda L_{idle}} \frac{(\lambda L_{idle})^j}{j!}$$

- $s(j)$: probability that j pkts arrive in a slot given success

$$s(j) = e^{\lambda L_{succ}} \frac{(\lambda L_{succ})^j}{j!}$$

- $c(j)$: probability that j pkts arrive in a slot given collision

$$c(j) = e^{\lambda L_{coll}} \frac{(\lambda L_{coll})^j}{j!}$$

- Given $N(t) = n$, # attempts at t $\sim Binomial(n, \beta_n)$
- Probability of an idle, a success and a collision slot

$$p_{idle,n} = (1 - \beta_n)^n$$

$$p_{succ,n} = n\beta_n(1 - \beta_n)^{n-1}$$

$$p_{coll,n} = 1 - p_{idle,n} - p_{succ,n}$$

- Equal probability of departure

$$P\left(D_i(t+1) = 1 \middle| N(t) = n, L(t+1) = L_{succ}, Q_i(t) > 0\right) = \frac{1}{n}$$

The Joint Queue Length Process

- The joint queue length process

$$\boldsymbol{Q}(t) := \big(Q_1(t), Q_2(t), \cdots, Q_M(t)\big)$$

is a DTMC

Theorem 1 *The DTMC* $\{\boldsymbol{Q}(t), t \geq 0\}$ *is positive recurrent if*

$$M\lambda < \min_{1 \leq n \leq M} \Theta_{sat,n}$$

where $\Theta_{sat,n}$ *is the mean aggregate throughput in packets/sec in a single cell consisting of* n *saturated nodes.*

Remarks: For large M ($M \geq 5$)

$$\min_{1 \leq n \leq M} \Theta_{sat,n} = \Theta_{sat,M}$$

- For large M, $\{\boldsymbol{Q}(t)\}$ is positive recurrent if $M\lambda < \Theta_{sat,M}$
- The stationary distribution exists
- The state space $\{0, 1, 2, \cdots\}^M$ is huge
- The model could be programmed into *ns-2*

Reduction of the State Space

A Decoupling Technique

- Define the state at channel slot boundary t as

$$\mathcal{X}(t) := \big(Q_1(t), \mathcal{M}(t)\big)$$

- $\mathcal{M}(t)$: # non-tagged node that are non-empty at time t

- The state space has reduced to

$$\{0, 1, 2, \cdots\} \times \{0, 1, \cdots, M-1\}$$

- $Q_*(t)$: # packets in the non-tagged queue at t from which
  a departure occurs at $t+1$

- To obtain stationary prob. of $\{\mathcal{X}(t)\}$ we need

$$q_{j,k} = P\Big(Q_*(t) = 1 \Big| Q_1(t) = j, \mathcal{M}(t) = k, Q_*(t) > 0\Big)$$

An Approximation

Approximation 0.1 ***(Conditional Independence)*** *[Sykas, Garetto]*

$$P\Big(Q_*(t) = 1 \mid Q_1(t) = j, \mathcal{M}(t) = k, Q_*(t) > 0\Big)$$

$$= P\Big(Q_*(t) = 1 \mid N(t), Q_*(t) > 0\Big) =: \tilde{q}_n$$

- The transition probabilities can be obtained now
- Observe that

$$\begin{aligned} \tilde{q}_n &= P\left(\tilde{Q}_1 = 1 \middle| \tilde{Q}_1 > 0, \tilde{N} = n\right) && \text{(by homogeneity)} \\ &= \frac{\tilde{\pi}(1, n-1)}{\sum_{j=1}^{\infty} \tilde{\pi}(j, n-1)} && \text{(by definition)} \end{aligned}$$

- An iterative method of solution for finite buffer

- Stationary probability that n nodes are non-empty at any random channel slot boundary

$$\begin{aligned}\tilde{p}_n &:= P\left(\tilde{N} = n\right) \\ &= P\left(\tilde{Q}_1 = 0, \tilde{\mathcal{M}} = n\right) + P\left(\tilde{Q}_1 > 0, \tilde{\mathcal{M}} = n - 1\right) \\ &= \tilde{\pi}(0, n) + \sum_{j=1}^{\infty} \tilde{\pi}(j, n-1) \qquad (1)\end{aligned}$$

Deriving Performance Measures

Collision Probability γ

$\mathcal{A}(t)$: # attempts up to time t

$\mathcal{C}(t)$: # collisions up to time t

$$\gamma := \lim_{t\to\infty} \frac{\mathcal{C}(t)}{\mathcal{A}(t)} \stackrel{a.s.}{=} \frac{\sum_{n=0}^{M} p_n E_n C}{\sum_{n=0}^{M} p_n E_n A} \approx \frac{\sum_{n=0}^{M} \tilde{p}_n E_n C}{\sum_{n=0}^{M} \tilde{p}_n E_n A}$$

$E_n A = n\beta_n$ and $E_n C = n\beta_n \left(1 - (1-\beta_n)^{n-1}\right)$

Aggregate Throughput Θ

$\mathcal{S}(t)$: # of successes by all nodes up to time t

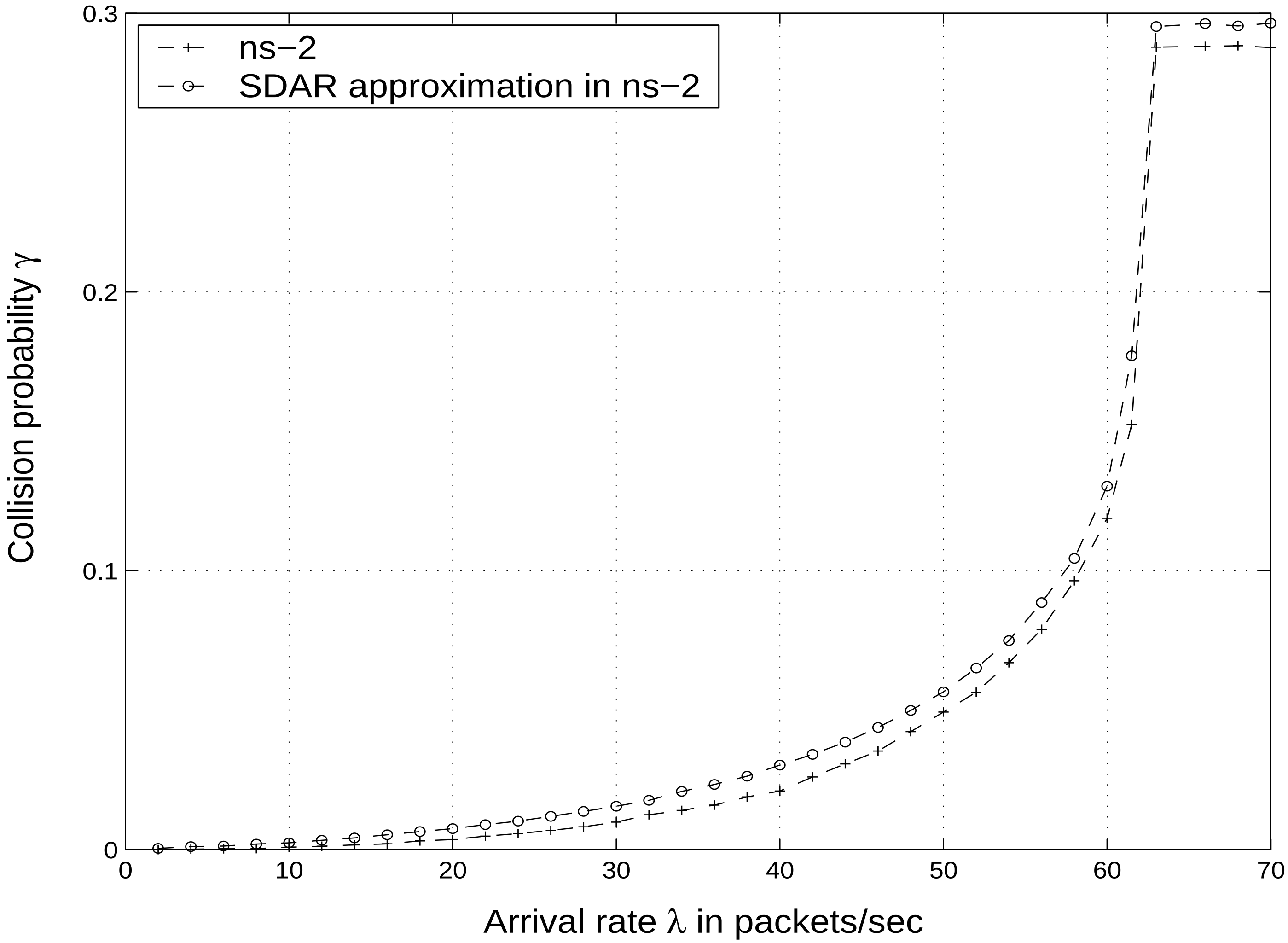
$$\Theta := \lim_{t\to\infty} \frac{\mathcal{S}(t)}{t} \stackrel{a.s.}{=} \frac{\sum_{n=0}^{M} p_n E_n S}{\sum_{n=0}^{M} p_n E_n L} \approx \frac{\sum_{n=0}^{M} \tilde{p}_n E_n S}{\sum_{n=0}^{M} \tilde{p}_n E_n L}$$

$E_n S = n\beta_n(1-\beta_n)^{n-1}$ and

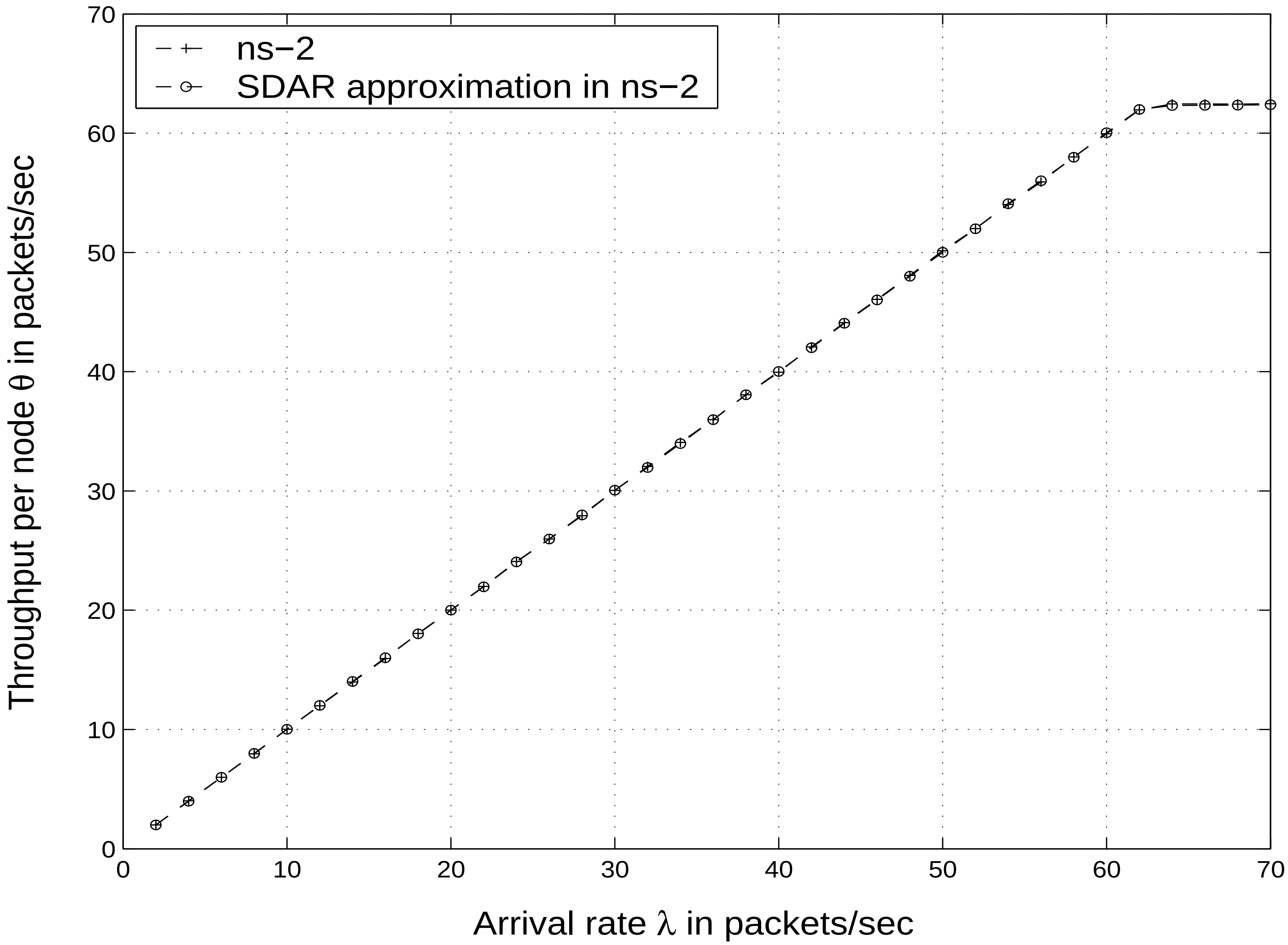
$E_n L = \sigma(1 + p_{coll,n}T_c + p_{succ,n}T_s)$

Numerical and Simulation Results

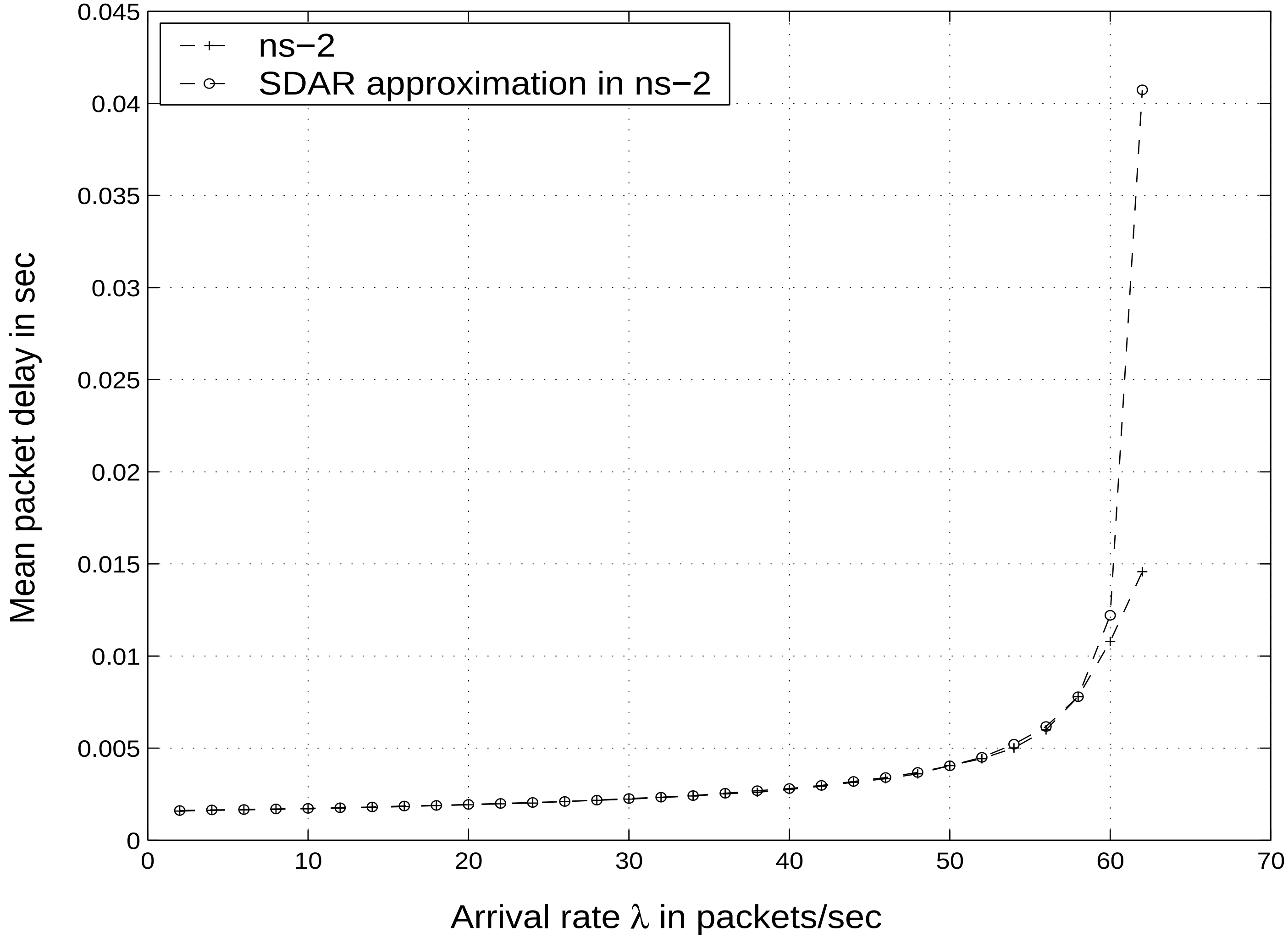
γ for Infinite Buffer Case



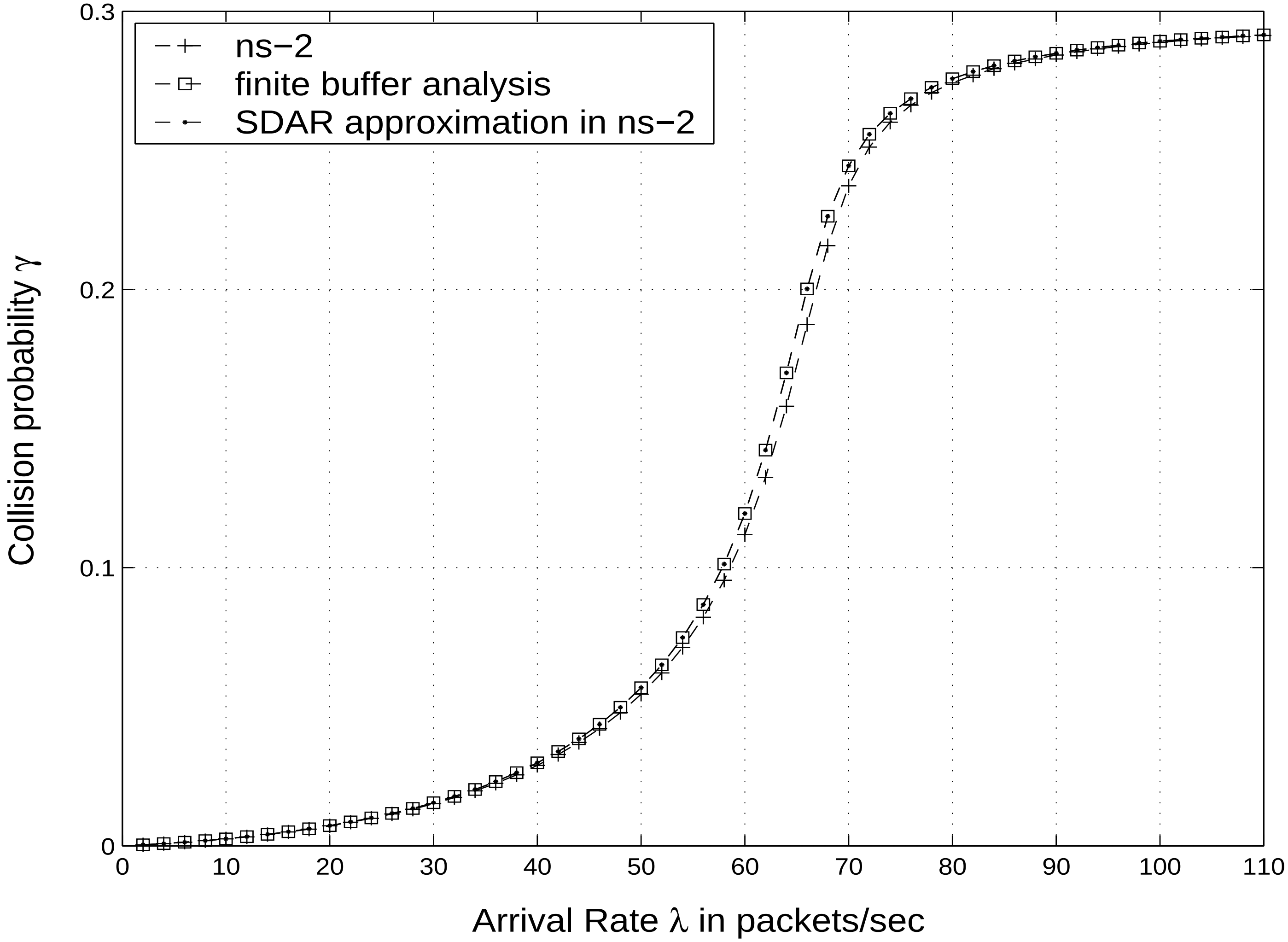
θ for Infinite Buffer Case



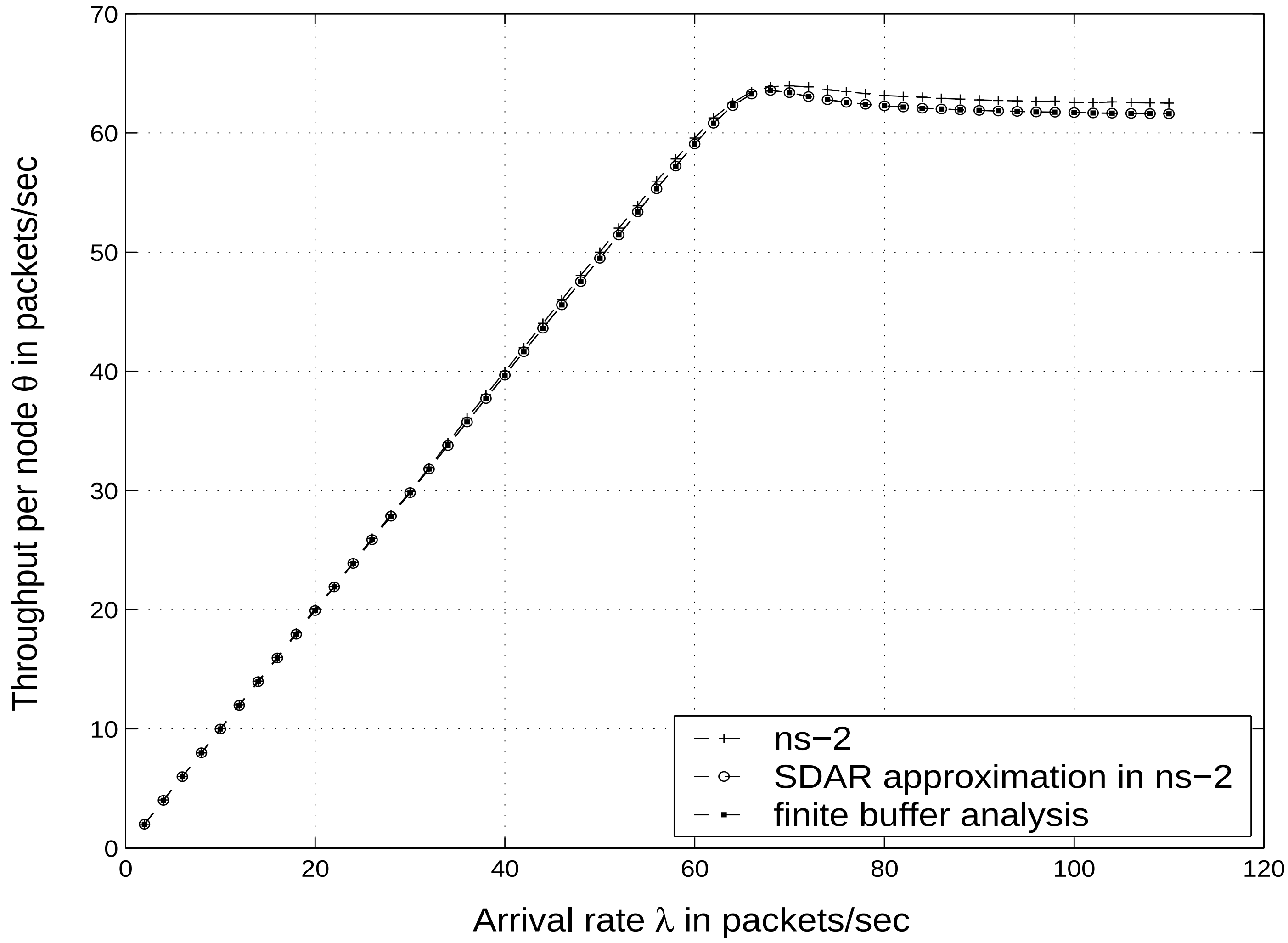
Mean Packet Delay for Infinite Buffer Case



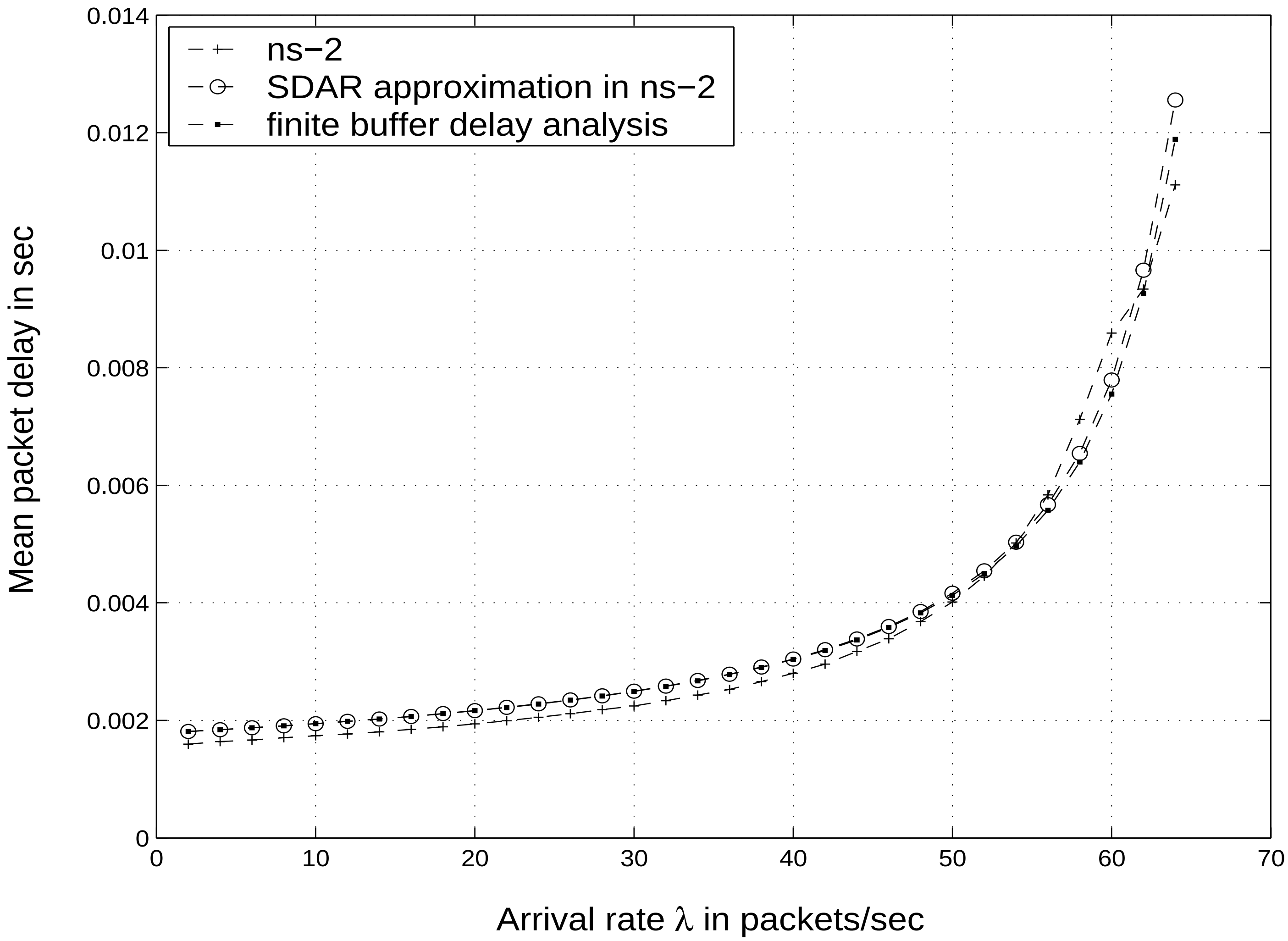
γ for Buffer Size 5



θ for Buffer Size 5



Mean Packet Delay for Buffer Size 5



Conclusions

- Modeled the network as a coupled queue system
- Studied its positive recurrence
- Developed a technique to reduce the state space
- Obtained accurate results
- An important theoretical question: Why the SDAR approximation works well?
- Future Work: Non-homogeneous nodes with unequal arrival rates, more general arrival processes

References

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