Distributed Scheduling for Multi-Hop Wireless Networks

Nachiket Sahasrabudhe , Joy Kuri

Centre for Electronics Design and Technology, Indian Institute of Science, Bangalore

Network Utility Maximization (NUM) Problem

- Given the end-to-end flows that are going to operate in the network, our aim is to maximize the aggregate network utility.

Network Model

- A network is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$.
- Multiple end-to-end multi-hop flows are present in the network.
- All source nodes are saturated.
- If x_f is the data-rate associated with a flow f, we associate a nondecreasing, concave utility function $U(x_f)$, with $U(0) = 0$, with every flow f in the network.

Interference in Wireless Scenario

- Activity on one link affects the activity on the other.
- Only a subset of links can be activated simultaneously.
- Link operates at effective link capacity which is lesser than the actual link capacity.
- K-hop interference model: $K + 1$ consecutive links interfere with each other.

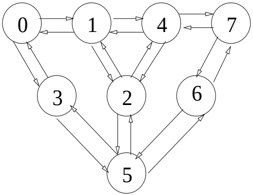


Figure: A wireless network

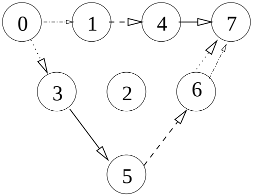


Figure: Independent set of links:
$I_1 = [(0,1),(6,7)]$, $I_2 = [(4,7),(3,5)]$, $I_3 = [(0,3),(6,7)]$, $I_4 = [(1,4),(5,6)]$

Primal Problem

- Associate a concave utility function $U_f(x_f)$, with each end-to-end flow.
- Network Utility Maximization (NUM) Problem:

$$\text{maximize: } f(\mathbf{x}) = \sum_{f \in \mathcal{F}} U(x_f)$$

$$\begin{aligned}
\text{Subject to: } \mathbf{A}\mathbf{y}_f &= \mathbf{u}_f, \forall f \in \mathcal{F} && (1)\\
\sum_{f \in \mathcal{F}} \mathbf{y}_f &\leq \mathbf{M}_l \mathbf{a} && (2)\\
\sum_{i=1}^{K} a_i &= 1 && (3)\\
\mathbf{x} &\geq \mathbf{0} && (4)
\end{aligned}$$

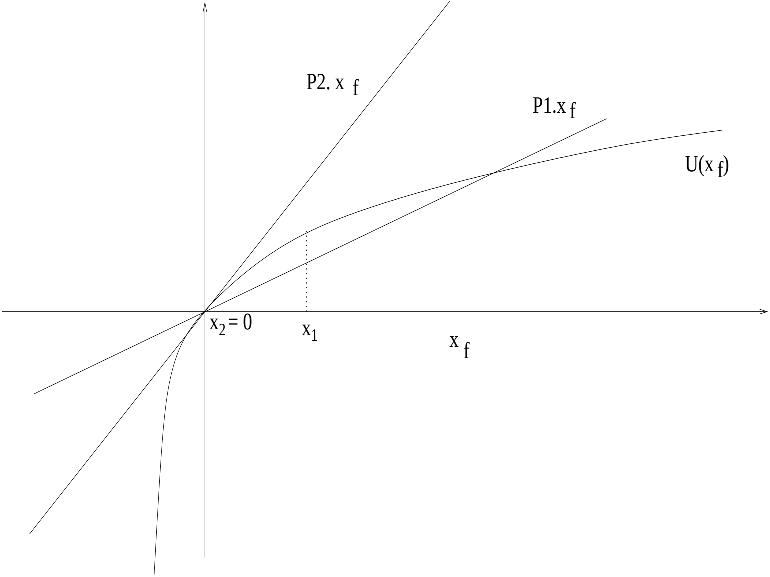
- Convex optimization problem with affine constraints: Duality gap is zero.

A solution to the congestion control and routing problem

- $$D_1(\mathbf{p}) = \max \sum_{f \in \mathcal{F}} (U(x_f) - p_R(f)x_f) \tag{5}$$
$$\text{subject to: } \mathbf{y}_f \geq \mathbf{0}, \forall f \in \mathcal{F}$$
$$x_f \geq 0, \forall f \in \mathcal{F}.$$
- The data is routed along the path for which the path price is minimum.
- Congestion control:

$$x_f = U'^{-1}(p_{R_{min}}(f)), \forall f \in \mathcal{F} \tag{6}$$

Utility Function



Optimal Schedule

- Optimal schedule **a**, maximizes $\mathbf{p}^T \mathbf{c}_{eff}$
-

$$D_2(\mathbf{p}) = \max(\mathbf{p}^{\mathbf{T}}\mathbf{M}_l\mathbf{a}) \tag{7}$$
$$\text{subject to: } \sum_{i=1}^{i=K} a_i = 1$$
$$\mathbf{a} \geq 0$$

- Columns of $\mathbf{M}_l$ represent the independent sets in the network.
- $M_{l_{ij}} = 1$ if link i is part of j^{th} independent set, else $M_{l_{ij}} = 0$.
- The fraction of time a j^{th} independent set is activated is given by a_j.

Optimal scheduling scheme

- The optimal schedule for a given price vector is maximum weighted independent set.
- Optimal scheduling algorithm needs a centralized entity that is aware of the network topology .
- Finding an optimal schedule is an NP-hard problem.

Distributed Greedy Heuristics

- *Key notion*: Schedule an independent set containing a maximum weighted link, rather than scheduling a maximum weighted independent set.
- *Greedy Algorithm*:
  1. Price dissemination over 2-hop neighbourhood.
  2. Identification of interfering links.
  3. Price sorting.
  4. Link scheduling.
  5. Price updation.

Solution of the Dual problem by ϵ-subgradient algorithm

- Price updation equation:

$$\mathbf{p} = \mathbf{p} - \delta \mathbf{h}_\epsilon(\mathbf{p}) \quad (8)$$

$$p_l[j+1] = (p_l[j] - \delta(c_l[j] - y_l[j]))^+ \quad (9)$$

Definition

Given a convex function $D(\mathbf{p}) : \Re^n \rightarrow \Re$ and $\epsilon \geq 0$, a vector $\mathbf{h}(\mathbf{p}_0) \in \Re^n$ is an ϵ-subgradient of $D(\mathbf{p})$ at point $\mathbf{p}_0 \in \Re^n$ if $D(\mathbf{p}) \geq D(\mathbf{p}_0) - \epsilon + (\mathbf{p} - \mathbf{p_0})^T \mathbf{h}(\mathbf{p}_0), \forall \mathbf{p} \in \Re^n$.

- $$\mathbf{h}_\epsilon(\mathbf{p}) = \mathbf{c}_\epsilon(\mathbf{p}) - \mathbf{y}(\mathbf{p}) \quad (10)$$
- $\epsilon = 0$ case represents a case of the optimal solution.

Degree of Suboptimality

- If we select an independent set in some arbitrary manner, can we guarantee that the solution of Network Utility Maximization problem will approach the optimal solution?

Results

Proposition

There exist an $0 < M < \infty$ *and* $j_0 < \infty$*, such that* $p_{max}[j] \leq M \; \forall j \geq j_0$ *,under any scheduling policy that schedules an independent set in each time slot.*

Proposition

Suppose at each iteration j *an* ϵ_j*-subgradient is used. If* $\epsilon_j \leq \epsilon_0 \; \forall j$ *or* $\lim_{j \to \infty} \epsilon_j = \epsilon_0$ *and* $||h(j)||_2 \leq H \; \forall j$*, then* ϵ*-subgradient algorithm converges within* $\delta H^2/2 + \epsilon_0$ *of the optimal value.*

p_{max} remains upper bounded

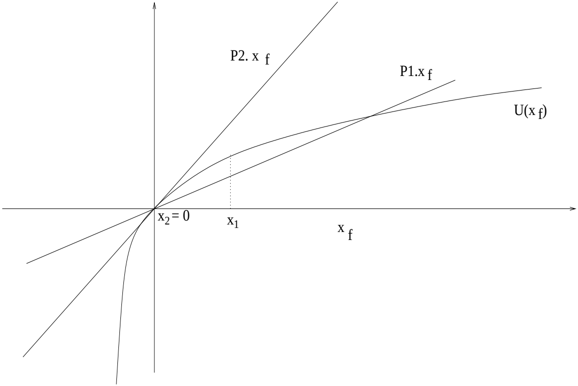


Figure: An example illustrating variation of optimal x_f under different values of S_{min}.

$$p_l[j+1] = (p_l[j] - \delta(c_l[j] - y_l[j]))^+ \qquad (11)$$

An example of interference degree $d_K(G)$, for $K = 2$

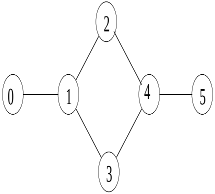


Figure: A network graph

- Set of links that interfere with link $l_{(1,2)}$:$\{l_{(0,1)}, l_{(1,3)}, l_{(2,4)}, l_{(3,4)}, l_{(4,5)}\}$.
- $d_K(l_{(1,2)}) = 2$.
- $d_K(G) = \max_{l \in \mathcal{L}} d_K(l) = 2$

Claim

Consider a line graph, with sufficiently large number of links L, each with capacity C. Then the path-price S satisfies the inequality

$$\frac{K+1}{K+1+C} \leq S \leq \frac{2K+1}{2K+1+C} \tag{12}$$

Proposition

Under the same set of assumptions for utility function, interference model and number of end-to-end flows, as for the claim above, in an arbitrary network graph $S \leq 1$ under any scheduling policy that schedules an independent set in each time slot.

Proposition

Any scheduling algorithm that schedules a set of non-interfering links at each time instant leads to ϵ-subgradient algorithm.

Proposition

The greedy scheduling policy leads to $LC\frac{d_K(G)-1}{d_K(G)}$ −subgradient.

Numerical Evaluation

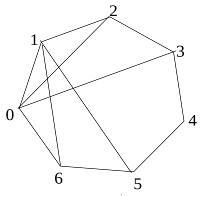


Figure: Network G_1. Flow is present between following pairs of nodes:$(0,4),(5,1),(3,6)$.

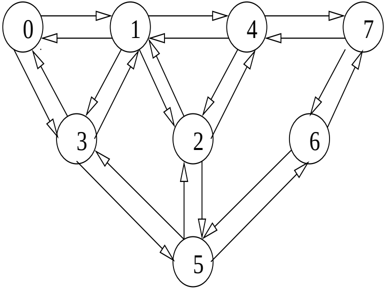


Figure: Network G_2. Flow is present between following pairs of nodes:$(0, 1), (5, 4), (6, 1)$.

Utility Plot

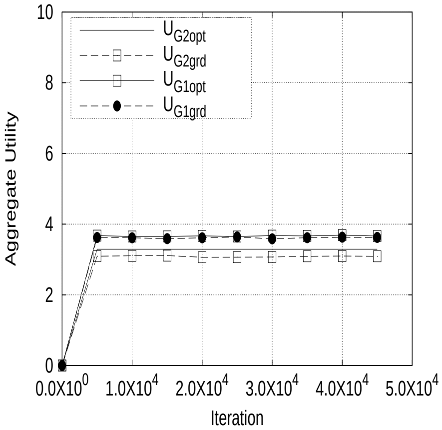


Figure: Utility function under optimal and greedy schedules for networks G_1 and G_2.

p_{max} variation

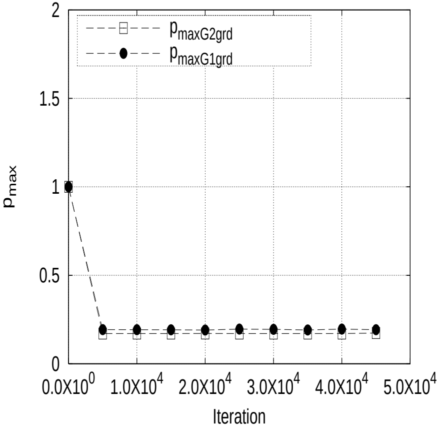


Figure: p_{max} remains bounded in networks G_1 and G_2

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