Improved Interference Diversity in Multicellular OFDMA Systems

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***Abstract*— Orthogonal frequency division multiple access (OFDMA) is becoming a popular technology choice for leading next generation wireless networks. The IEEE 802.16e based systems are the first standardized wireless networks which have incorporated OFDMA. We investigate the use of interference diversity in IEEE 802.16e based networks by considering subchannel formation in different cells. We propose a method for quantifying the interference diversity using suitable arguments and results. Based on these results, a new method for forming subchannels in an 802.16e system is presented.**

keywords—OFDM, Interference Diversity, WiMAX

I. INTRODUCTION

Typical deployment of wide-area wireless systems uses a cellular approach so that the spectrum can be used efficiently. Most cellular systems are employing reuse factors close to 1 (originally pioneered by IS-95 CDMA systems) so as to satisfy the concerns of spectrum regulators. Hence, tight frequency reuse will be a major selling point for a technology which is a candidate for Broadband Cellular Wireless (BCW) [1, 2] systems. To satisfy this requirement, WiMAX has specified subchannelization methods which take this challenge into account. In multi-cellular OFDMA systems with tight frequency reuse, there is no inherent protection from the ensuing co-channel interference (CCI). Hence, to address this problem, the mapping of subcarriers to subchannels is done differently for the different cells in the neighborhood. The mandatory Diversity Subcarrier Method (DSM) wherein the subchannel typically contains non contiguous subcarriers is expected to be used in most of the cases. Let us consider a frequency reuse scenario as an example since this is potential deployment scenario in WiMAX which is enabled by the Partially Utilized Sub-Channels (PUSC) method.

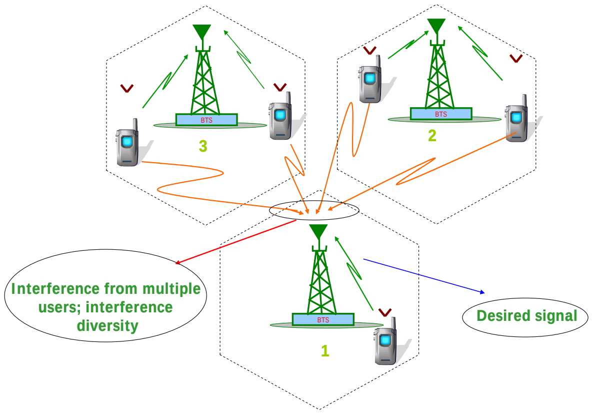

Figure 1. Interference in multicellular operation

Let the desired user be in cell 1 and let us consider the uplink transmission in that cell. Let us assume that slow power control is also in effect as this is a standard operation in most cellular systems. The mobile will use one or more subchannels for transmitting data to the Base Station (BS) in the cell. At the same time, other mobiles in the neighboring co-channel cells/sectors could also be transmitting to their respective BS'. Due to frequency reuse, there is a potential for severe CCI as some of the users in the neighboring cells could be reusing the subcarriers used by the desired mobile in cell 1. However, note that it is likely that all the interferers are not going to be at the same distance and location from the desired BS receiver. This scenario is illustrated in Fig. 1. This fact is used in the subchannelization in the different cells. The subcarriers that constitute a subchannel are also determined by the cell identification number (called UL_perm_base in 802.16e) which is different for the cells in the neighborhood. On the downlink, it is called DL_perm_base. Thus, the interference seen in the subcarriers of a subchannel in our desired cell is likely to come from different subchannels in the neighboring cells because of the difference in the subchannel definitions in the neighborhood. This means that, it is likely that the interference in a subchannel is likely to come from different users in the neighborhood and these users are likely to be in different locations and fading conditions. This can potentially

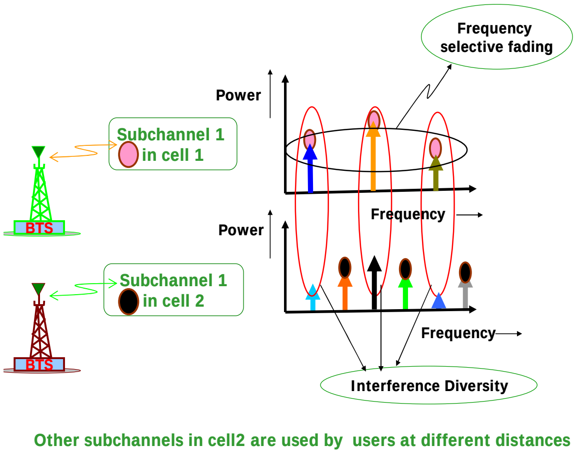

Figure 2. Illustration of interference diversity in multicelluar systems

lead to an interference diversity effect, i.e. the interference powers on the subcarriers constituting a subchannel can be different.

In Fig. 2**,** for simplicity consider 3 subcarriers belonging to subchannel 1 in cell 1. In cell 2, the same subcarriers are part of different subchannels which are used by different users of cell 2 that are at different locations from the BS in cell 1. The common subcarriers in both the subchannels are covered by the vertical ovals. The heights of the arrows are used to illustrate the power received on these subcarriers at BS of cell 1, i.e., it is the interference power as seen at the BS. Note that the difference in heights of the subcarrier arrows could be due to the different locations of the users. In Fig. 2, we have illustrated the representative positions of the 3 subcarriers which constitute the subchannel 1 in cell 2. Note the difference in positions of the subcarriers from cell 1 as this is likely to happen when different DL_perm_base values are used.

Effectively, the Signal to Interference plus Noise Ratio (SINR) is frequency selective due to the multi path channel conditions and the varying interference conditions on the subcarriers in a subchannel. The Bit Interleaved Coded Modulation (BICM) techniques are effective in such conditions and can result in good error performance as they can leverage the frequency diversity. It is unlikely that the Fully Utilized Sub Channel (FUSC) method will be used in the initial rollout of WiMAX systems except for users close to the BS. However, for operation under tight frequency reuse conditions, as for example in 1/3 reuse, which is very likely, the interference diversity effects are important as the subchannelization in PUSC depends on DL_perm_base. Note that in PUSC, the interference will come not from other sectors in the same cell but there will be significant interference from neighboring cells which use the same carrier frequency.

II. INTERFERECNE DIVERSITY INSPECTION

We have considered a WiMAX system with 5 MHz bandwidth and a total of 512 subcarriers. Under these conditions, there are a total of 15and 17 subchannels in the downlink and uplink respectively. A subchannel typically is a grouping of 24 subcarriers whose positions are distributed in the frequency space. A 3 sector per cell deployment is considered. A sector in the cell is assigned 5 subchannels in the DL. In the UL, sector 0, 1 and 2 are allocated 6, 6 and 5 subchannels respectively.

TABLE I. INTERFERENCE DIVERSITY OF AN 802.16E BASED SYSTEM ON THE DOWNLINK

Reference Subchannel	**Number of Subcarriers contributing interference to reference subchannel**														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	5	0	1	1	1	3	0	3	1	2	3	1	1	0	2
2	2	2	0	2	2	2	5	1	1	0	2	2	1	2	0
3	1	2	4	0	1	1	2	4	1	1	0	2	1	1	3
4	1	1	1	3	0	1	1	1	5	2	3	0	3	2	0
5	0	0	0	1	5	1	4	2	2	2	1	0	0	2	4

TABLE II. INTERFERENCE DIVERSITY OF AN 802.16E BASED SYSTEM ON THE UPLINK

Reference Subchannel	**Number of Subcarriers contributing interference to reference subchannel**																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	4	4	4	0	0	4	0	0	0	0	0	0	8	0	0	0
2	0	0	4	0	4	0	0	4	0	0	4	0	0	0	8	0	0
3	0	0	0	4	0	4	0	4	0	0	0	4	0	0	0	8	0
4	0	0	0	0	4	0	0	0	4	0	0	0	4	4	0	0	8
5	4	0	0	0	0	4	0	0	4	4	0	0	0	4	4	0	0
6	0	0	0	4	0	0	4	0	0	4	4	0	0	0	4	4	0

The effect of interference diversity on the downlink for the different subchannels in a particular reference cell is captured in Table I. In this table, we consider 5 subchannels in a sector of a reference cell with a particular value of DL_perm_base and we consider a neighboring cell which reuses the same carrier frequency. The comparison carried out in Table I is for the DL_perm_base values in the WiMAX draft [3].

In each subchannel, there are 24 subcarriers and in the neighboring cell, typically these subcarriers are assigned to different subchannels. In Table I, in each row the distribution of the 24 subcarriers across the 15 subchannels in the neighboring cell is shown. It can be seen from Table I that for each of the reference subchannels, the interfering subcarriers are distributed across 12-13 subchannels in the neighboring cell. Extensive experiments performed with other values of the permutation base indicate a similar trend on an average case. A few reference subchannels received fewer interference contributing subchannels while others received a higher number of interference contributing subchannels. This is how interference diversity manifests itself in the downlink PUSC allocation

In the uplink, the interference diversity manifests in a slightly different form due to the uplink PUSC allocation wherein 4 consecutive subcarriers are considered as one unit. From Table II, it is seen that 5-6 different subchannels contribute interference to a particular subchannel in the reference cell. The comparison carried out in Table II is for the UL_perm_base values in the WiMAX draft [4].

Extensive experiments performed with other values of the permutation base indicate a similar trend on an average case. A few reference subchannels received fewer interference contributing subchannels while others received a higher number of interference contributing subchannels. Similar trends have been noticed in other experiments as well. Thus, interference diversity which ultimately implies receiving different levels of interference (from a statistical standpoint) on the subcarriers in a subchannel is achieved in WiMAX by assigning subcarriers to subchannels in different cells in a permuted predetermined order.

III. MEASURING INTERFERENCE DIVERSITY

A. *Parameters and Considerations*

Since interference diversity is important in OFDMA systems, it is important to quantify the amount of interference diversity that is present under various scenarios. The idea is based on the heuristic that in a reference subchannel in a cell, if there are more interference contributing subchannels from a neighboring co-channel cell, then the interference diversity experienced is higher than in the case where there are a lesser number of interference contributing subchannels. Interference diversity is measured for a reference subchannel in a reference cell. The following parameters are taken into account to design the metric

1. The number of interfering subchannels with respect to a reference subchannel in a reference cell.
2. The subcarrier distribution from the interfering subchannels with respect to a reference subchannel in a reference cell.

The following considerations and goals are taken into account to design the metric

1. The interference diversity per reference subchannel is to be quantified by the metric $x=\{x \mid 0 \leq x \leq 1\}$.
2. The best interference diversity is to be attained when $x=1$
3. The worst interference diversity is to be attained when $x=0$.
4. If $0 \leq x_1<x_2 \leq 1$, then x_2 indicates better interference diversity over x_1.
5. The best interference diversity is expected when all the subchannels in a cell contribute interfering subcarriers towards a reference subchannel in a reference cell.
6. The worst interference diversity is expected when only one subchannel in a cell contributes interfering subcarriers towards a reference subchannel in a reference cell.
7. The number of subchannels contributing interference should be an indicator of the interference diversity metric. A greater number of interfering subchannels should indicate better interference diversity and vice versa.
8. A large number of contributing interfering subcarriers from a subchannel should be reflected by a proportional decrease in the interference diversity metric.

B. *Interference Diversity Metric (IDM)*

The following steps are used to obtain the proposed interference diversity metric

1. Let *m* be the total number of subchannels in the sector of the reference cell.
2. Let *n* be the total number of subchannels in the reference cell.
3. Let *threshold* be the number of subcarriers, above which the contribution from interfering subcarriers in an interfering subchannel towards a reference subchannel is not considered.
4. Determine the number of contributing interference subcarriers from each subchannel in the neighboring co-channel cell towards the reference subchannel in the reference cell.

TABLE III. ILLUSTRATION OF METRIC FUNCTION

Reference Subchannel	**Number of Subcarriers contributing interference to reference subchannel**															**Total weight**
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	4	7	15/15
2	1	1	1	1	1	1	1	1	1	1	1	1	1	11	0	13/15
3	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	12/15
4	5	5	5	5	4	0	0	0	0	0	0	0	0	0	0	5/15
5	11	11	2	0	0	0	0	0	0	0	0	0	0	0	0	1/15

5. Let *count* be the number of subchannels from which the number of interference contributing subcarriers is less than or equal to *threshold*.
6. The value of x for the reference subchannel in the reference cell is *count*/*n*.

The value of *threshold* chosen for the experiments here is *n*/2. The choice of the value of *threshold* is subjective and is set suitably when an interfering subchannel is considered to be a dominant interferer. The metric for measuring interference diversity is explained with the following example. Consider a downlink PUSC subchannel with 24 subcarriers (refer Table III). In the example considered there are 15 subchannels from a neighboring cell which can potentially contribute interference to the subchannel in the reference cell. Consider any row in Table III which tabulates the number of interfering subcarriers to the reference subchannel in the example reference cell. A weight 1 is assigned for every non zero entry less than or equal to 15/2 (half the number of subchannels in a cell) and weight 0 to every other entry in this example. The metric is the sum of all the weights. It is noted that a weight 1 is assigned only to those columns of the row which have a non-zero entry. The emphasis for assigning a weight 1, is in having a non zero entry in the column and not on the size of the entry in the column. However large the value in the column gets, it still adds up only one unit to the total weight. It is observed that when a particular column or a set of columns receive large entries, the other columns tend to get a zero entry because the total interfering subcarriers in the row is fixed (24 in this case).

A threshold (15/2 here) is placed to prevent dominant interfering subchannels from adding up to the weight. Moreover, larger the count of columns with non zero entries, greater is its weight. This is based on the notion that if there are more interfering sources, then the metric should increase correspondingly. Also, for two cases where there are same numbers of interfering subchannels but a particular subchannel contributes much more than others, then that situation is addressed by giving a zero weight to the column entries with a weight greater than threshold. The maximum value of metric in this particular case is 15/15.

Note that Table III illustrates the function of the metric using a hypothetical example. In the ideal case all the columns should be occupied by non-zero entries less than or equal to the threshold value. Since there are 15 interfering subchannels (for Down link), the weight of row 1 in Table III is calculated to be the sum of unit weight assigned to all the row entries less than or equal to 15/2, the threshold assigned. The weight of row 1 is found to be 15/15 = 1. The weight for row 2 is found to be 13/15. Clearly, reference subchannel 2 has lower interference diversity than reference subchannel 1 and gets a lower weight. The weight of row 3 is found to be 12/15. If the two interferers in column 1, row 3 is removed and added up to column 2, row 3; the new weight of row 3 is expected to decrease to indicate the new diversity. The weight of modified row 3 is 11/15 and agrees to our expectation. The weight of row 4 is 5/15. If each of the 5 interferes in row 4, column 1 were to be removed and added one each to column 6 to 10 of row 4, the modified weight of row 4 is 9/15 as expected. The weight of row 5 is 1/15. The entries in the first and second column of row 5 are above the threshold limit of 15/2 and are set to 0, since they are considered as dominant interferers.

For the uplink (refer Table II), the value of the parameter *n* in the IDM measuring algorithm is set to 6, which is the maximum number of subchannels contributing interference to the reference subchannel.

IV. IMPROVING INTERFERENCE DIVERSITY

Our objective is to create a new method to form subchannels such that it improves the IDM when compared to the WiMAX standard. Consider N subcarriers in a cell. The new method uses a Pseudo Random Number Generator as an input to a shuffle algorithm to get a random ordering of the numbers 0 to N-1 which represents physical subcarrier positions. Subchannels are formed by grouping blocks of L

TABLE IV. SATTOLO'S SHUFFLE ILLUSTRATED

Range	**Random**	**Swap | Result**
		1 2 3 4 5 |
1–5	3	1 2 **5** 4 | **3**
1–4	1	**4** 2 5 | **1** 3
1–3	2	4 **5** | **2** 1 3
1–2	1	5 | **4** 2 1 3

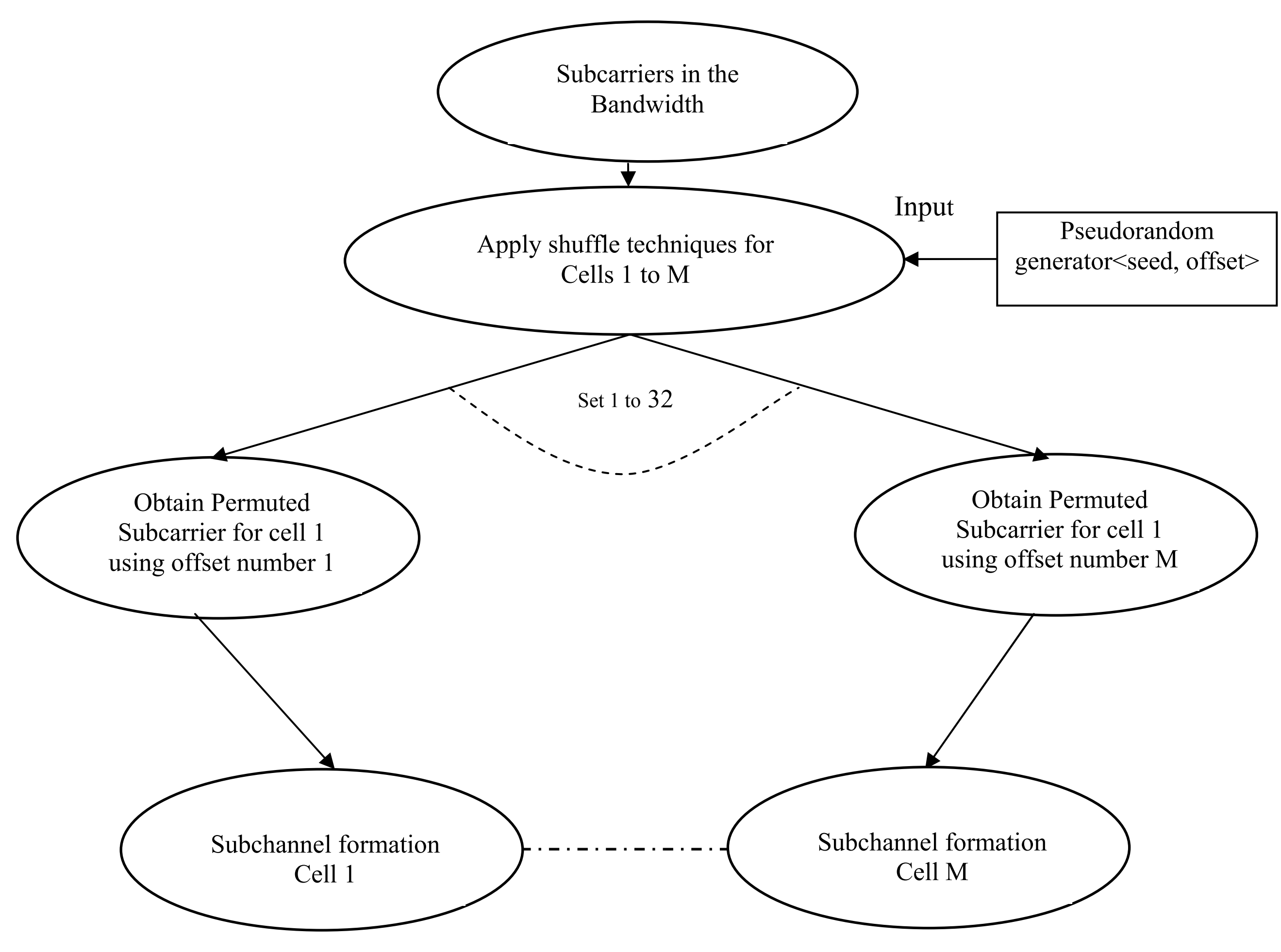

Figure 3. Illustration of the steps in the new method for forming subchannels

subcarriers from the shuffled set. Since IDM is improved by having little commonality in physical subcarrier positions amongst subchannels in different cells, a different shuffle is obtained for another cell reusing the same spectrum. Thus, subchannels in any two cells are likely to contain very different subcarriers leading to better IDM values. Define *seed* to be the initialization value for the PRNG and *offset* to be the iteration index of the PRNG from which pseudo random number generation should continue to generate N physical subcarrier re-ordering using the shuffle algorithm, for a given cell. The PRNG is run continuously with initial value *seed* to generate 32 different shuffle sets of N subcarriers each for the 32 cells. There are hence 32 offsets. Each base station in its respective cell broadcasts its tuple <seed, offset number> so that mobile stations are able to retrieve the physical ordering of the subcarriers in the cell, given the tuple. Hence, the PRNG input to the shuffle algorithm for any particular cell is likely to be very different from any other cell because the PRNG which acts as a random shuffle position to the shuffle algorithm runs with the same initial seed and different offset.

In particular, the proposed method uses Sattolo's shuffle algorithm [5] to permute the physical subcarrier numbers. Let *SI* be the set of physical data subcarrier indices used in the WiMAX standard. For the proposed scheme the same sub carrier index set *SI* is used for assigning physical data subcarriers but is specified by a different permutation. An example that permutes 5 numbers using Sattolo's shuffle is illustrated in Table IV. Here 'Random' is the random number generated. For our case, the pseudo random generator used is Mersenne Twister (MT19937) [6], a uniformly equi-distributed pseudo random number generator. Multiplicative and linear congruence generators [7] may be used to generate

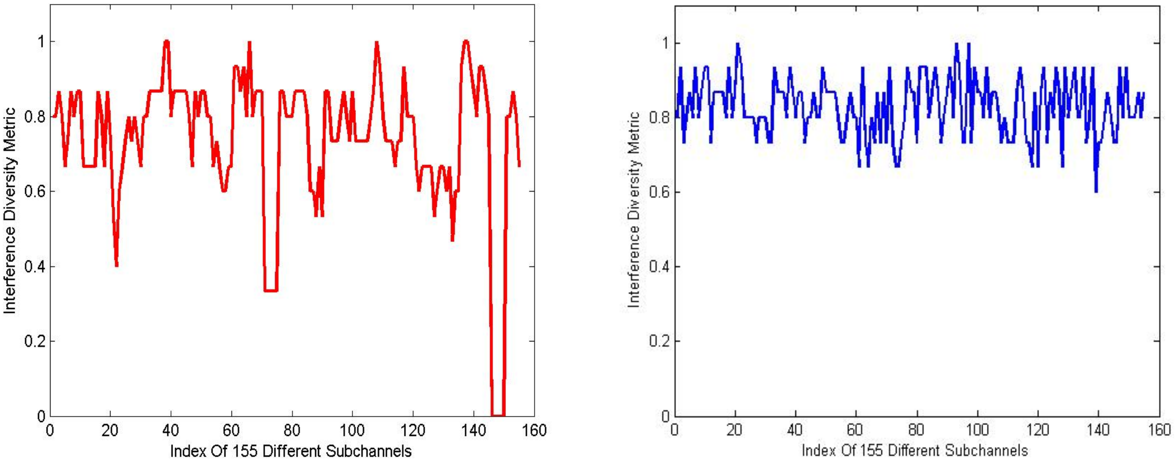

Figure 4. (a) WiMAX standard's metric for Downlink (b) Proposed method's metric for Downlink. Reference cell 0, reference subchannel 1 to 5 in both figure

full period pseudo random sequences. However, due to the poor quality of random numbers generated [8] a 'good' pseudo random number generator namely MT19937 is used. MT19937 passes both the Diehard test for randomness [9] and the NIST test suite [10, 11]. The choice of the Sattolo's shuffle follows from the requirement to produce pseudo random permutations in a desired interval.

A. Parameters and Considerations

We propose a new method for forming subchannels and the IDM is calculated for this new method. The most significant improvement is in improving the average IDM.

The following steps are carried out in the proposed method to create a new permutation base for the downlink

1. Use Sattolo's shuffle algorithm to generate 32 sets of 360 physical subcarrier numbers in the range $0 \le subcarrier\_number < 360$. The uniformly equi-distributed pseudo random number generator, MT19937 is used to determine the random shuffle position. The real number version, 53 bit precision output mode of MT19937 is used to *output* real numbers in the interval $0 \le output < 1$. Next, $\lfloor output * 360 \rfloor$ is computed to get pseudo random integers in the interval $0 \le output < 360$ which is used as the random shuffle position in Sattolo's shuffle algorithm to get 360 permuted physical subcarriers in the interval $0 \le subcarrier\_number < 360$. This is done for all the 32 cells.

2. Let the sets obtained be P_0 to P_{31}, each with 360 distinct permuted physical subcarriers in the range $0 \le subcarrier\_number < 360$. The permutation P_i is assigned to ID_Cell_i (for all $0 \le i < 32$). Let the physical ordering of the 360 permuted subcarrier in an arbitrary ID_Cell_i be denoted as $S_i = \{s_{i,0}, s_{i,1}, \ldots, s_{i,359}\}$.

3. Let $sch_{i,1}$ to $sch_{i,15}$ be the 15 subchannels in ID_Cell_i ($0 \le i < 32$). For each ID_Cell_i, the physical subcarriers are allocated as follows:

   a) All subchannels are allocated 24 subcarriers, in the order which they appear in set S_i. I.e. $sch_{i,1} = \{s_{i,0}, \ldots s_{i,23}\}, \ldots, sch_{i,2} = \{s_{i,24}$ to $s_{i,47}\}, \ldots, sch_{i,15} = \{s_{i,336}$ to $s_{i,359}\}$.

V. INTERFERENCE DIVERSITY RESULTS

The superiority of this new method over existing methods is shown using the IDM. We have considered a 5 MHz OFDMA system with 512 subcarriers and we have used the method in the WiMAX system for comparison. We use the IDM for measuring the performance of the methods.

To capture the interference diversity trends, we have found the mean value of the interference diversity and the standard deviation of these values and compare our method with that of WiMAX standard. The plots are consistent for different reference cells.

In Fig. 4, we compare the IDM performance of the new method with that of the existing method proposed in WiMAX. The interference diversity metric is evaluated for each reference subchannel in the reference cell. Subchannels (1-5) in sector 0 are chosen to be reference subchannels. The DL_perm_base value of the interfering cell is varied from 1 to 31 while the reference cell's value is '0'. Thus, there are 155 values of IDM which are obtained in this experiment and plotted in Fig. 4. Similar results were obtained with different values of the DL_perm_base for the reference cell. The x abscissa in Fig. 4 is the index value of the 155 different subchannels against which the IDM is measured. The y ordinate gives the corresponding IDM values.

We have considered various DL_perm_base values for the reference cell and considered a 3 sector deployment as in earlier results. We measured the IDM for interference from various candidate interfering cells with different permutation

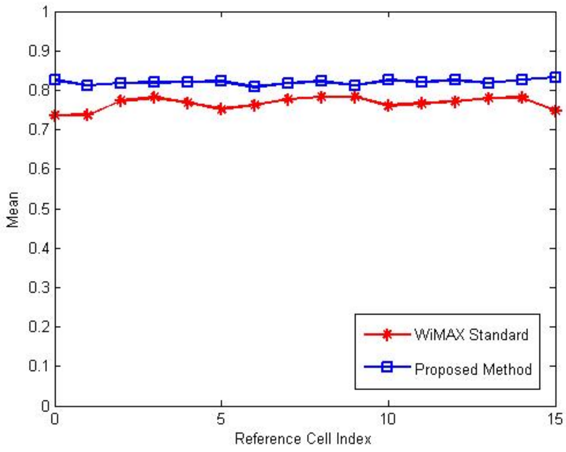

Figure 5. Reference Cell Index Vs Mean

base values on the downlink (DL_perm_base). Fig.4 (a) refers to the output generated by the interference diversity metric applied on the WiMAX standard's algorithm and Fig.4 (b) refers to the output generated by the IDM obtained using the proposed method. From Fig.4 (a) it is observed that a number of interfering co-channels have poor interference diversity with respect to the reference subchannels in the reference cell. The interference diversity metric with respect to certain reference subchannels measured for the WiMAX standard in Fig.4 (a) is seen to have values as low as 0.4 to 0.0. From Fig.4 (b) it is observed that the interfering co-channels have good interference diversity with respect to the reference subchannels in the reference cell. The value of the IDM for the proposed method is observed to average around 0.8.

Further, there are no co-channels with low IDM values as present for the WiMAX standard. Reference subchannels 1-5 in reference cells 0 to 15 are used for comparison in downlink in Fig. 5. The arithmetic mean of the interference diversity metric is shown in Fig. 5; on the downlink for the WiMAX standard algorithm and the proposed method. A higher arithmetic mean clearly indicates better interference diversity with respect to the reference subchannels in the reference cell. It is seen that the proposed method consistently has a better mean IDM for the downlink when compared to the WiMAX standard.

Let *mean* denote the arithmetic mean of the IDM, for a given reference cell. The *negative standard deviation* (*nsd*) for a given reference cell is the standard deviation of all the IDM below the *mean* IDM in that cell. It is used to measure the variations in IDM below the arithmetic mean for different reference cells.

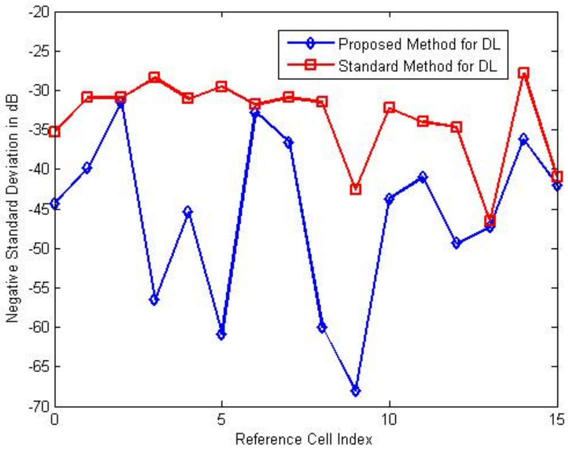

Figure 6. Reference Cell Index Vs Negative Standard Deviation for Downlink

The negative standard deviation for Downlink (Reference cells 0 to 15) is shown in Fig. 6. It is clear from the plot in Fig. 6 that the proposed method has a lower *Negative standard deviation* when compared to the WiMAX standard and hence better interference diversity than that of the WiMAX Standard.

VI. CONCLUSION

The first contribution of this paper is to propose a new method to measure interference diversity for 802.16e based WiMAX systems. The second contribution is to propose a new method to improve the interference diversity over the WiMAX standard. The proposed IDM is used to compare the interference diversity of the new method with the WiMAX standard.

The IDM measurement for an 802.16e based WiMAX systems, for 5 MHz PUSC deployment with 512 FFT point for the downlink was carried out. The proposed method was found to have better interference diversity with respect to the WiMAX standard. The IDM measurement for an 802.16e based WiMAX systems, for 5 MHz PUSC deployment with 512 FFT point for the uplink was also carried out. For the uplink, contiguous subcarriers are grouped to help with channel estimation. An approach similar to the proposed downlink permutation base was used to determine a new permutation base for the uplink. The interference diversity of the proposed system was found to be just at par with the WiMAX standard. The grouping of subchannels results in a smaller set of subcarrier group indices that needs to be permuted. The extend to which one can de-correlate smaller sets of indices using the shuffle is rather limited.

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