Amplify Based Double-Differential Modulation for Cooperative Communications

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Abstract— **In this paper, we propose double-differential (DD) modulation for amplify-and-forward (AAF) cooperative communications system. The proposed system works very well in the presence of carrier offsets without any channel knowledge, where the previously proposed differential AAF cooperative system breaks down. We have derived the approximate symbol error rate (SER) of the proposed system. In addition, numerical power allocation is proposed to further improve the performance of the system. The proposed cooperative system outperforms the same rate trained cooperative system and conventional direct transmission based DD system.**

I. INTRODUCTION

In the last few years, it has been proved that cooperative communication is a promising technique for providing transmit diversity and improved capacity without a physical antenna array [1]. The cooperation between the users helps them to share the system resources to improve the quality of service. The cooperative transmission converts the single-input single-output (SISO) systems into a virtual MIMO system by adding additional space dimension to the transmitting terminals [1], [2].

Various cooperation protocols for wireless systems were proposed in [3]. In the most basic decode-and-forward (DAF) protocol, a user (source) needs to select another user who agrees to relay its data to the destination. The source sends information to the relay and destination as well. The relay decodes the data sent by the source and retransmits the decoded data to the destination. Hence, the destination has two received replicas of the same data and the quality of reception is expected to improve. However, even at high signal to noise ratio (SNR) the relay cannot decode the data perfectly and relays erroneous data to the destination. This causes error floor in the performance, and, hence, the DAF protocol cannot achieve full diversity in its pure form and needs some intelligence added to the relay or destination for improving the performance. This may cause increase in the cost and power consumption at the relays and destination. Another form of basic relaying protocol is the amplify-and-forward (AAF) protocol. In this scheme, the relay simply amplifies the received data without decoding it and retransmits the amplified data toward the destination. The destination now has two different copies of the same data and applies signal processing techniques to estimate the original data. The AAF protocol does not need any advanced signal processing at the relay or destination contrary to the DAF protocol and generally performs better than the DAF protocol particularly at higher SNRs.

It is normally assumed that the destination and relay have perfect knowledge of the channel. However, in practice, it is difficult to estimate the channel gains accurately when the channel remains stationary for small number of symbol durations. In order to avoid channel estimation, differential modulation can be implemented in cooperative communication systems [4], [5], [6], [7]. However, if there is a carrier offset due to the mismatch between the transmit and receive oscillators or relative motion of the receiver and transmitter, the channel does *not* remain stationary over two consecutive time-periods, which is an assumption in differential coding. In such cases, the performance of the differential scheme degrades substantially. This is an important practical problem for cooperative wireless communications, which has not yet been given much consideration. A conventional and popular solution of removing the effect of carrier offset from the received data is to use a training based carrier offset estimator at the receiver. This estimated carrier offset is used to compensate the received data before the actual decoding [8], [9], [10]. However, this method reduces the effective data rate. Moreover, the compensation error increases with time and may introduce an error floor in the system performance [11].

In this paper, our main contributions are: 1) We propose double-differential (DD) modulation for AAF cooperative wireless communications to overcome the fast variation of the channel because of carrier offsets. 2) We derive an upper bound for the SER for DD modulation with AAF protocol (DDAAF). 3) Based on the upper bound of the SER expressions, we determine the numerical power allocation for the DD cooperative systems.

The rest of this paper is organized as follows: In Section II, the system model and DD modulation for a single-input single-output links are discussed. Section III implements DD modulation in the AAF cooperative communications. The SER performance analysis of DD modulation with AAF protocol is performed in Section IV. A training based cooperative system is implemented in Section V. In Section VI, the analytical and simulation results are discussed and details of power allocation for minimizing the upper bound of SER of DDAAF cooperative system is provided. Section VII contains some conclusions.

This work was supported by the Research Council of Norway project 176773/S10 called OptiMO, which belongs to the VERDIKT program.

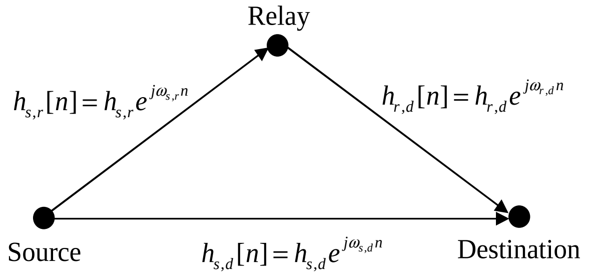


Fig. 1. Cooperative system with carrier offsets.

II. SYSTEM MODEL AND DD MODULATION

We consider a basic cooperative communication system, which consists of one source s, one relay r, and one destination d terminal as shown in Fig. 1. Each of them can either transmit or receive a signal at a time. The transmission of the data from source to destination terminal is furnished in two phases. In the first phase, the source broadcasts data to the destination and the relay. The relay amplifies the received data and retransmits it to the destination, in the second phase. To avoid interference, the source and the relay use orthogonal channels for transmission [3]. For ease of presentation we assume that in both phases, the source and relay transmit stream of data through time-division multiplexing (TDMA). In the TDMA scheme, the source has to remain silent in the second phase in order to maintain the orthogonality between the transmissions. However, in the frequency-division multiplexing (FDMA) or the code-division multiplexing (CDMA) schemes, the source and the relay can transmit at the same time.

The time varying channel between link p and q, i.e., $h_{p,q}[n]$ is modeled as $h_{p,q}[n] = h_{p,q}e^{\jmath\omega_{p,q}n}$, where $(p,q) \in \{(s,r),(r,d),(s,d)\}$, $h_{p,q}$ is a flat fading channel coefficient which remains constant over at least *three* time-intervals, and $\omega_{p,q}$ is random (normalized) carrier offset which also remains constant over at least *three* time-intervals. However, due to the phase term $e^{\jmath\omega_{p,q}n}$, $h_{p,q}[n]$ varies with time n even if $h_{p,q}$ and $\omega_{p,q}$ are constant. The random offsets $\omega_{p,q}$ are assumed to be uniformly distributed over $[-\pi,\pi>$, however, in general, there is no restriction over the probability distribution of the offsets and they could be arbitrarily distributed.

Let $z[n]$ denote the symbols belonging to the unit-norm M-PSK constellation Ξ to be transmitted at the time n. In DD modulation, the transmitted signal $v[n]$ is obtained from $z[n]$ as shown in Fig. 2 (a):

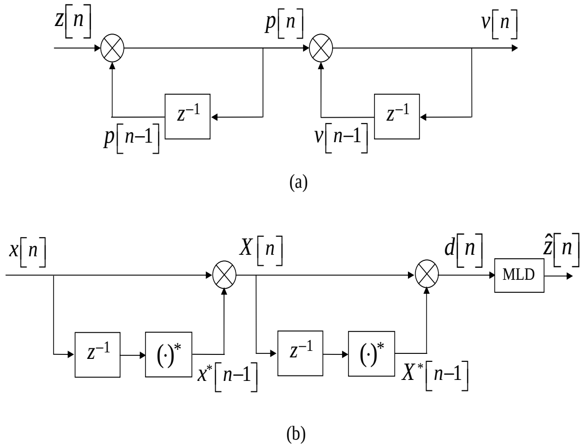
$$\begin{aligned} p[n] &= p[n-1]\, z[n]\,, \\ v[n] &= v[n-1]\, p[n]\,, \quad n = 2,3,\ldots, \end{aligned} \tag{1}$$

with $|v[0]| = |v[1]| = 1$. As $|z[n]| = 1$ for the unit-norm M-PSK symbols, it follows from (1) that $|v[n]| = |p[n]| = 1, \forall n \geq 0$. We consider a flat fading SISO channel with carrier offset described by

$$x[n] = \sqrt{\rho}h[n]v[n] + e[n]\,, \quad n = 0,1,\ldots, \tag{2}$$

where $h[n] = he^{\jmath\omega n}$, $x[n]$ is the received signal, ρ is the transmitted signal power, h is the channel gain, $e[n]$ is complex-valued additive white Gaussian noise (AWGN), and $\omega \in [-\pi,\pi>$ is an unknown frequency offset. The receiver makes a decision variable, $d[n] = X[n]X^*[n-1]$, where $X[n] = x[n]\,x^*[n-1]$ as shown in Fig. 2 (b). The maximum likelihood (ML) optimal decoding of $z[n]$ is performed in the following way [14, Eq. (15)]:

$$\hat{z}[n] = \underset{z\in\Xi}{\operatorname{argmax}} \operatorname{Re}\{X[n]X^*[n-1]z^*\}\,. \tag{3}$$



(a)

(b)

Fig. 2. Double-differential (a) encoder and (b) decoder.

III. DOUBLE-DIFFERENTIAL MODULATION FOR AAF COOPERATIVE COMMUNICATION SYSTEM

If we use DD modulation in the cooperative communication system, the data received during the *first phase* at the destination is

$$x_{s,d}[n] = \sqrt{P_1}h_{s,d}[n]v[n] + e_{s,d}[n]\,, \tag{4}$$

and at the relay is

$$x_{s,r}[n] = \sqrt{P_1}h_{s,r}[n]v[n] + e_{s,r}[n]\,, \tag{5}$$

where P_1 is the power transmitted by the source, $h_{s,d}[n] = h_{s,d}e^{\jmath\omega_{s,d}n}$, $h_{s,r}[n] = h_{s,r}e^{\jmath\omega_{s,r}n}$, $h_{s,d}$ and $h_{s,r}$ are the flat fading channel gains, and $\omega_{s,d}$ and $\omega_{s,r}$ are the carrier offsets between source and destination, and source and relay, respectively, $e_{s,d}[n]$ and $e_{s,r}[n]$ are AWGN noise of the two links, $n = 0,1,\ldots,N-1$, and N is the number of time-intervals used for transmission. The relay amplifies the received data of (5) and retransmits during *second phase*, such that the received signal by the destination in the *second phase* is:

$$x_{r,d}[m] = \sqrt{\hat{P}_2}h_{r,d}[m]x_{s,r}[m] + e_{r,d}[m]\,, \tag{6}$$

where $h_{r,d}[m] = h_{r,d}e^{\jmath\omega_{r,d}m}$, $h_{r,d}$ is the flat fading channel gain, $\omega_{r,d}$ is the carrier offset between relay and destination, $e_{r,d}[m]$ is the AWGN noise, $m = 0,1,\ldots,N-1$, and $\hat{P}_2$ is amplification factor to maintain a constant average transmission power during the second phase and defined as

$$\hat{P}_2 = \frac{P_2}{P_1\,|h_{s,r}|^2 + \sigma^2}, \tag{7}$$

where P_2 is the average power transmitted by relay. It is assumed that $P_1 + P_2 = P$, where P is the total transmitted power from the source and relay over the two phases. Next, we propose the following maximal ratio combining (MRC) [12] based scheme for a DDAAF receiver

$$\begin{aligned} d\left[k\right] &= \alpha_1 (x_{s,d}\left[n\right] x_{s,d}^*\left[n-1\right])(x_{s,d}\left[n-1\right] x_{s,d}^*\left[n-2\right])^* \\ &+ \alpha_2 (x_{r,d}\left[m\right] x_{r,d}^*\left[m-1\right])(x_{r,d}\left[m-1\right] x_{r,d}^*\left[m-2\right])^*, \end{aligned} \quad (8)$$

where $k=n=m$, i.e., the data received by the destination during the same time interval with respect to the beginning of the each phase is combined, and α_1 and α_2 are given as

$$\alpha_1 = \frac{1}{\left(2P_1 \left|h_{s,d}\right|^2 + \sigma^2\right)\sigma^2}, \quad (9)$$

and

$$\alpha_2 = \frac{\left(P_1 \left|h_{s,r}\right|^2 + \sigma^2\right)^2}{\kappa}, \quad (10)$$

where

$$\begin{aligned} \kappa = &\Big\{ 2P_1 P_2^2 \left|h_{r,d}\right|^4 \left|h_{s,r}\right|^2 \sigma^2 + 2P_1 P_2 \\ &\times \left(P_1 \sigma_{s,r}^2 + \sigma^2\right) \left|h_{r,d}\right|^2 \left|h_{s,r}\right|^2 \sigma^2 + P_2^2 \left|h_{r,d}\right|^4 \sigma^4 + 2P_2 \\ &\times \left(P_1 \sigma_{s,r}^2 + \sigma^2\right) \left|h_{r,d}\right|^2 \sigma^4 + \left(P_1 \sigma_{s,r}^2 + \sigma^2\right)^2 \sigma^4 \Big\}. \end{aligned}$$

In (8) α_1^{-1} and α_2^{-1} are the noise variances in the decision variables $x_{s,d}\left[n-1\right] x_{s,d}^*\left[n-2\right]$ and $x_{r,d}\left[m-1\right] x_{r,d}^*\left[m-2\right]$, respectively. However, as we intend to use DD modulation, the destination and relay are not expected to have knowledge of the channel gains, therefore, we can emulate the MRC by replacing the channel coefficients $\left|h_{s,r}\right|^2$, $\left|h_{r,d}\right|^2$, and $\left|h_{s,d}\right|^2$ by their variances $\sigma_{s,r}^2$, $\sigma_{r,d}^2$, and $\sigma_{s,d}^2$, respectively, in (7), (9), and (10). Then the data is decoded as

$$\hat{z}\left[n\right] = \arg\max_{z \in \Xi} \mathrm{Re}\left\{d\left[k\right] z^*\right\}. \quad (11)$$

IV. SER PERFORMANCE ANALYSIS

The emulated maximum ratio combining (EMRC) obtained by replacing the squared absolute value of the channel gains by their variances in (8) will perform poorer to the ideal MRC scheme given by (8), (9), and (9) [12]. For the simplicity, we assume that the instantaneous SNR of the EMRC scheme is

$$\gamma \approx \gamma_{s,d} + \gamma_{s,r,d}, \quad (12)$$

where $\gamma_{s,d}$ and $\gamma_{s,r,d}$ are the instantaneous SNRs of the direct link between source and destination, and cooperative link between source and destination through relay, respectively. This assumption is justified by the simulation results in Subsection VI-A as the EMRC scheme performs very close to the ideal MRC scheme. If M-PSK modulation is used with instantaneous SNR γ, the conditional probability of symbol error for double-differential modulation can be written with the help of [18, Eq. (13)] and [17, Eq. (11b)] as

$$P_s\left\{h_{s,d}, h_{s,r}, h_{r,d}\right\} = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \exp\left(-\gamma\, \xi_{\frac{\pi}{M}}\left(\psi\right)\right) d\psi, \quad (13)$$

where $\xi_{\frac{\pi}{M}}\left(\psi\right) = \frac{\sin^2 \frac{\pi}{M}}{\sin^2\psi + \sin^2\left(\psi + \frac{\pi}{M}\right)}$. Averaging (13) over the channel gains we get the following SER

$$P_s = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_{\gamma_{s,d}}\left(\xi_{\frac{\pi}{M}}\left(\psi\right)\right) M_{\gamma_{s,r,d}}\left(\xi_{\frac{\pi}{M}}\left(\psi\right)\right) d\psi, \quad (14)$$

where $M_\gamma(\cdot)$ denotes the moment generating function (MGF) of γ. For independent Rayleigh fading channels, $|h_{s,d}|^2$, $|h_{r,d}|^2$, and $|h_{s,r}|^2$ are independent exponential random variables with parameters $1/\sigma_{s,d}^2$, $1/\sigma_{r,d}^2$, and $1/\sigma_{s,r}^2$, respectively. The SNR of the cooperative link between source and destination through relay (s,r,d) under DD modulation can be obtained by substituting (5) and (7) into (6), and finding average signal and noise power of $X_{r,d}[m] = x_{r,d}[m] x_{r,d}^*[m-1]$ as

$$\gamma_{s,r,d} = \frac{P_1^2 P_2^2 \left|h_{s,r}\right|^4 \left|h_{r,d}\right|^4}{\kappa}. \quad (15)$$

Assuming $P_1 \left|h_{s,r}\right|^2 + P_2 \left|h_{r,d}\right|^2 >> \sigma^2$, then using the result of the MGF of harmonic mean of two random variables in [19, Eq. (7)], MGF of the decision variable of DD modulation in [18, Eqs. (20) and (22)], and after some manipulation it can be shown from (15) that the MGF of $\gamma_{s,r,d}$ can be written as

$$\begin{aligned} M_{\gamma_{s,r,d}}\left(\xi_{\frac{\pi}{M}}\left(\psi\right)\right) &= \frac{256 P_1 P_2 \sigma_{s,r}^2 \sigma_{r,d}^2 \sigma^4 \exp\left(-\frac{\xi_{\frac{\pi}{M}}(\psi)}{4}\right)}{3\beta^2(\xi_{\frac{\pi}{M}}\left(\psi\right))} \\ &\times \left\{ \frac{16\left(P_1 \sigma_{s,r}^2 + P_2 \sigma_{r,d}^2\right)\sigma^2}{\beta(\xi_{\frac{\pi}{M}}\left(\psi\right))} {}_2F_1\left(3, \frac{3}{2}; \frac{5}{2}; \delta(\xi_{\frac{\pi}{M}}\left(\psi\right))\right) \right. \\ &\left. + {}_2F_1\left(2, \frac{1}{2}; \frac{5}{2}; \delta(\xi_{\frac{\pi}{M}}\left(\psi\right))\right) \right\}, \end{aligned} \quad (16)$$

where

$$\begin{aligned} \beta(\xi_{\frac{\pi}{M}}\left(\psi\right)) &= 4\sigma^2 \left(\sqrt{P_1}\sigma_{s,r} + \sqrt{P_2}\sigma_{r,d}\right)^2 \\ &+ \xi_{\frac{\pi}{M}}\left(\psi\right) P_1 P_2 \sigma_{s,r}^2 \sigma_{r,d}^2, \end{aligned} \quad (17)$$

$$\begin{aligned} \delta(\xi_{\frac{\pi}{M}}\left(\psi\right)) &= \frac{1}{\beta(\xi_{\frac{\pi}{M}}\left(\psi\right))} \left\{ 4\sigma^2 \left(\sqrt{P_1}\sigma_{s,r} - \sqrt{P_2}\sigma_{r,d}\right)^2 \right. \\ &\left. + \xi_{\frac{\pi}{M}}\left(\psi\right) P_1 P_2 \sigma_{s,r}^2 \sigma_{r,d}^2 \right\}, \end{aligned} \quad (18)$$

and ${}_2F_1\left(\cdot, \cdot; \cdot; \cdot\right)$ is the Gauss Hypergeometric Function [20, Eq. (15.1.1)]. The MGF of $\gamma_{s,d}$ can be written as [18]

$$M_{\gamma_{s,d}}\left(\xi_{\frac{\pi}{M}}\left(\psi\right)\right) = \frac{\exp\left(-\frac{\xi_{\frac{\pi}{M}}(\psi)}{4}\right)}{1 + \alpha_{s,d}(\xi_{\frac{\pi}{M}}\left(\psi\right))}, \quad (19)$$

where $\alpha_{s,d}(\xi_{\frac{\pi}{M}}\left(\psi\right)) = \xi_{\frac{\pi}{M}}\left(\psi\right) P_1 \sigma_{s,d}^2 / \left(2\sigma^2\right)$.

Lemma *1:* The SER of AAF cooperative communications system with DDMPSK modulation can be upper bounded as

$$\begin{aligned} P_s \leq &\frac{2048 P_1 P_2 \sigma_{s,r}^2 \sigma_{r,d}^2 \sigma^6 \exp\left(-\frac{\sin^2 \frac{\pi}{M}}{4}\right)}{3\kappa(\frac{\pi}{M})} \\ &\times \left\{ \frac{32\left(P_1 \sigma_{s,r}^2 + P_2 \sigma_{r,d}^2\right)\sigma^2}{\beta(\frac{\pi}{M})} {}_2F_1\left(3, \frac{3}{2}; \frac{5}{2}; \delta(\frac{\pi}{M})\right) \right. \\ &\left. + {}_2F_1\left(2, \frac{1}{2}; \frac{5}{2}; \delta(\frac{\pi}{M})\right) \right\}, \end{aligned} \quad (20)$$

where

$$\begin{aligned} \beta(\frac{\pi}{M}) &= 8\sigma^2 \left(\sqrt{P_1}\sigma_{s,r} + \sqrt{P_2}\sigma_{r,d}\right)^2 \\ &+ P_1 P_2 \sigma_{s,r}^2 \sigma_{r,d}^2 \sin^2(\frac{\pi}{M}), \end{aligned}$$

$$\delta\left(\frac{\pi}{M}\right)=\frac{1}{\beta\left(\frac{\pi}{M}\right)}\left\{8\sigma^2\left(\sqrt{P_1}\sigma_{s,r}-\sqrt{P_2}\sigma_{r,d}\right)^2 + \sin^2(\frac{\pi}{M})P_1P_2\sigma_{s,r}^2\sigma_{r,d}^2\right\},$$

and $\kappa(\frac{\pi}{M})=\left(2\sigma^2+P_1\sigma_{s,d}^2\sin^2\left(\frac{\pi}{M}\right)\right)\beta^2\left(\frac{\pi}{M}\right)$.

Proof: As $0\leq\sin\left(\psi+\frac{\pi}{M}\right),\sin\psi\leq 1$, therefore, setting $\sin\left(\psi+\frac{\pi}{M}\right)=\sin\psi=1$ in (14), we can obtain the upper bound of (20). ■

V. IMPLEMENTATION OF TRAINING BASED COOPERATIVE SYSTEM

In this section, we will show how to implement a trained amplify based cooperative system for comparison with the proposed DDAAF system. Let us assume that the trained decoder at the destination utilizes the *two* initialization symbols as training data, and estimates the carrier offsets and channels using the following maximum likelihood estimators [21, Eqs. (9.7.27) and (9.7.28)]:

$$\hat{\omega}=\arg\left\{x[1]x^*[0]\right\},\quad \hat{h}=\frac{1}{2}\left(x[0]+\exp(-\jmath 2\pi\hat{\omega})x[1]\right), \tag{21}$$

where $\arg\{\cdot\}$ provides angle of the complex scalar and $(\cdot)^*$ stands for the complex conjugate. [21, Eqs. (9.7.27) and (9.7.28)] are proposed for an $n_t\times N$ space-time block code (STBC) in a MIMO system, where n_t is the number of transmit antennas and N is the time dimension. However, we are working with a cooperative system containing SISO links. Therefore, we use $n_t=N=1$ in [21, Eqs. (9.7.27) and (9.7.28)] for obtaining (21). In the trained system, the symbols $z[n]$ are directly transmitted in the space without any differential encoding. Therefore, the received data equations for such a system can be obtained by replacing $v[n]$ by $z[n]$ in (4), (5), and (6). Let us also assume that $z[0]=z[1]=1$. The receiver at the destination makes the following MRC based decision variable [12]

$$\begin{aligned}d[k]=&\frac{\hat{h}_{s,d}^*}{\sigma^2}\exp(-\jmath 2\pi\hat{\omega}_{s,d})y_{s,d}[n]\\&+\frac{\hat{h}_{s,r,d}^*}{E_N}\exp(-\jmath 2\pi\hat{\omega}_{s,r,d})y_{r,d}[m],\quad k=n=m,\end{aligned} \tag{22}$$

where $h_{s,r,d}$ is the effective channel over the cooperative link (s,r,d), $\omega_{s,r,d}$ is the effective carrier offset introduced by the cooperative link, and $y_{s,d}[n]$ and $y_{r,d}[m]$ are the data received due to the direct transmission and relayed transmission, respectively, and E_N is the total noise power in $y_{r,d}[m]$, which is given by

$$E_N=\frac{\left(P_1\sigma_{s,r}^2+P_2|h_{r,d}|^2+\sigma^2\right)\sigma^2}{P_1\sigma_{s,r}^2+\sigma^2}. \tag{23}$$

From (23), it can be seen that E_N contains $|h_{r,d}|^2$. However, it is difficult to estimate $h_{r,d}$ separately as it can be seen from (6), that the relay transmits an *amplified* version of the *received* signal corresponding to the *training data* transmitted by the source. As the channel statistics vary far more slowly than the channel coefficients, we can assume that the destination has a perfect knowledge of $\sigma_{r,d}^2$. Therefore, the trained decoder can obtain the decision variable by replacing $|h_{r,d}|^2$ by $\sigma_{r,d}^2$ in (23). In addition, it can also be assumed that relay and destination has perfect knowledge about $\sigma_{s,r}^2,\sigma^2,P_1$, and P_2.

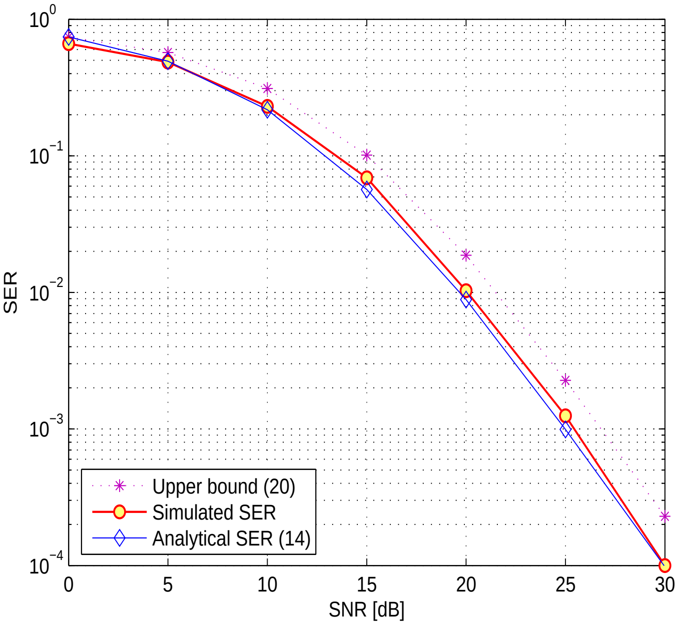


Fig. 3. Analytical and experimental performance of double-differential cooperative system with amplify-and-forward protocol.

VI. ANALYTICAL AND EXPERIMENTAL PERFORMANCE EVALUATION OF DDAAF SYSTEM

All simulations are performed with the following assumptions: All links are assumed to be Rayleigh distributed, which remain constant over at least *three* consecutive time intervals. Moreover, all the links are assumed to be perturbed by different random carrier offsets randomly distributed in the range of $[-\pi,\pi>$, which also remain constant over at least *three* consecutive time intervals. The symbols drawn from unit norm QPSK constellation are used for cooperative transmission. All simulation results are obtained for 10^5 channel realizations.

A. Analytical and Experimental Performance

Fig. 3 shows the analytical and experimental performance of the proposed DDDAF based cooperative scheme with random carrier offsets. We have plotted the upper bound of (20) for QPSK constellation, $P_1/P=P_2/P=0.5$ and $\sigma_{s,d}^2=\sigma_{s,r}^2=\sigma_{r,d}^2=1$. The simulation results of the proposed scheme are also shown under the same conditions. From Fig. 3, it is seen that the upper bound (20) is tight and remains consistent at approximately 1.5 dB from the simulated SER. We have also plotted the analytical SER obtained in (14). It can be seen from Fig. 3 that the experimental results follow closely the analytical results. Hence, this justifies the assumption taken in (12).

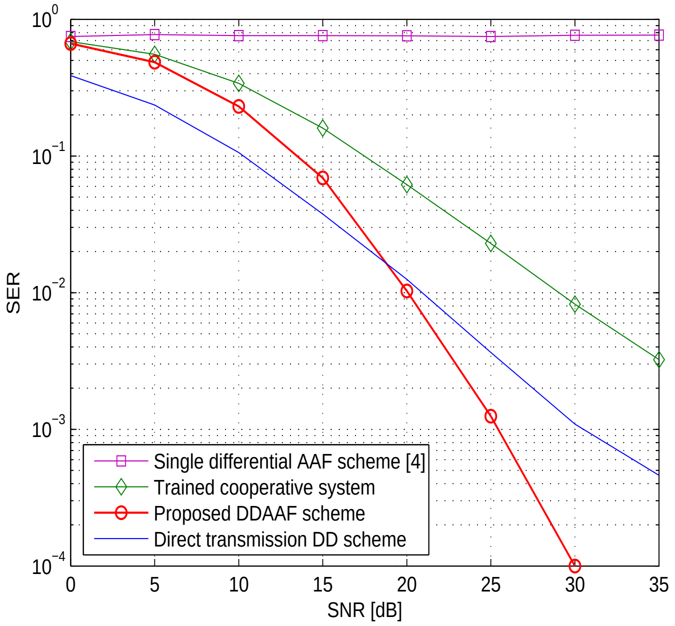


Fig. 4. Double-differential cooperative system with amplify-and-forward protocol and $P_1/P = P_2/P = 0.5$ and $\sigma^2_{s,d} = \sigma^2_{s,r} = \sigma^2_{r,d} = 1$.

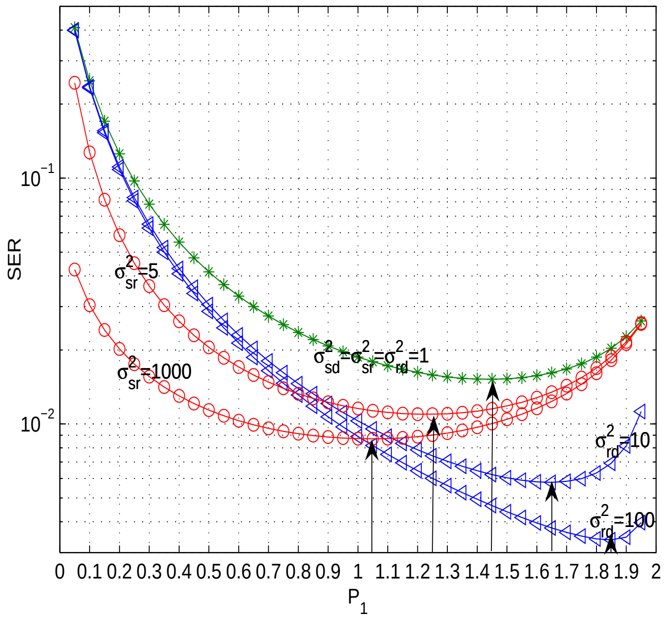


Fig. 5. Numerical power distribution by (20) in the case, when all links are available, $-\circ-$ for $\sigma^2_{s,d} = \sigma^2_{r,d} = 1$, $-\triangleleft-$ for $\sigma^2_{s,d} = \sigma^2_{s,r} = 1$, and $-*-$ for $\sigma^2_{s,d} = \sigma^2_{s,r} = \sigma^2_{r,d} = 1$.

B. Comparisons with Trained Cooperative System of Section V*, Direct DD Transmission, and Differential Cooperative System [4]*

Fig. 4 shows the comparison of the performance of the proposed DDAAF based cooperative scheme with the trained cooperative system developed in Section V, DD direct transmission, and previously proposed single differential amplify-and-forward cooperative scheme [4]. The cooperative schemes use QPSK constellation and direct DD transmission works with BPSK constellation such that all schemes utilize same rate of 1 bit/sec/Hz. From Fig. 4, it is seen that with all links of same quality, i.e., $\sigma^2_{s,d} = \sigma^2_{s,r} = \sigma^2_{r,d} = 1$, the proposed DDAAF scheme outperforms the direct DD transmission at the SNRs higher than 18 dB, with $P_1 = P_2 = 0.5P$. However, at the lower SNRs the proposed scheme performs poorer to the direct transmission scheme because the relay not only amplifies the signal but also amplifies the received noise, which is considerable in the low SNR range. It can be seen from Fig. 4 that the proposed scheme has higher diversity as compared to the direct transmission scheme and a performance gain of about 5 dB is observed at SER=10^{-3}. Moreover, the proposed cooperative DDAAF scheme has a prominent benefit over the direct transmission DD scheme that the later suffers a heavy penalty in the terms of performance if the channel between the source and destination is bad. However, the proposed cooperative DDAAF scheme can always achieve performance gain as compared to the direct transmission scheme by power distribution over the links as shown in the Subection VI-C. Further, It can be seen that the proposed DDAAF system outperforms the same rate trained cooperative system of Section V for all SNR values. From Fig. 4, it can also be observed that there is a collapse in the performance of the same rate (QPSK) conventional differential scheme [4], because of the random carrier offsets.

C. Power Allocation for the DDAAF Cooperative System

From (20), it can be seen that the SER of the DD cooperative system with AAF protocol depends non-linearly upon P_1 and P_2. Hence, if the total transmit power $P = P_1 + P_2$ is fixed, it is possible to find a power allocation for the source and relay terminals which minimizes the upper bound of the SER. Unfortunately, it is very difficult to find a closed-form solution of the minimum SER power distribution for DDAAF system. Therefore, we have used numerical method for finding the power distribution which minimizes the SER. Fig. 5 shows the power distribution plots for the case when all three links are available. All SER values are calculated from (20) with power constraint $P_1 + P_2 = P = 2$ and SNR = 20dB. It can be seen when all the links are of same quality the uniform power distribution does not provide the minimum SER, however, the power distribution of $P_1 = 1.45$ and $P_2 = 0.55$ minimizes the SER. An approximately uniform power distribution $P_1 = 1.05$ and $P_2 = 0.95$ minimizes the SER when the link between the source and the relay is very good: $\sigma^2_{s,r} = 1000$. Furthermore, it can be seen from Fig. 5 that the proposed DDAAF scheme generally tends to increase the source power to minimize the SER. Fig. 6 shows the performance of DDAAF scheme with different power distributions. The minimum SER power distribution $P_1 = 0.825P$ and $P_2 = 0.175P$ is found at SNR = 20dB by numerically minimizing (20) with power constraint $P = P_1 + P_2$. The simulations are performed with $\sigma^2_{s,d} = \sigma^2_{s,r} = 1$ and $\sigma^2_{r,d} = 10$. It can be seen from Fig. 6, that DDAAF scheme with minimum SER power distribution outperforms the DDAAF schemes with power distribution of

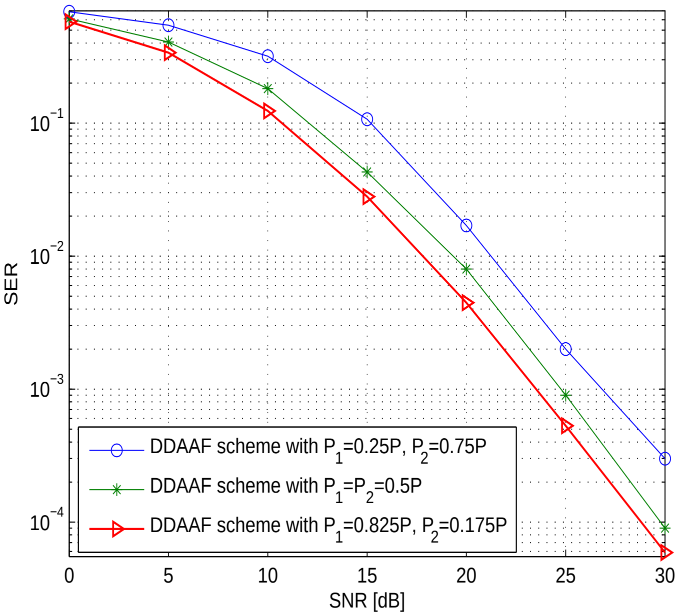


Fig. 6. DDAAF cooperative system with different power distributions and $\sigma^2_{s,r} = 1, \sigma^2_{s,d} = 1$, and $\sigma^2_{r,d} = 10$.

$P_1 = .25P, P_2 = 0.75P$ and uniform power distribution.

VII. CONCLUSIONS

We have implemented the double-differential modulation in cooperative communication system with amplify-and-forward protocol to avoid the problem of carrier offsets. Our scheme performs well in practical situations, where the conventional differential modulation scheme fails. With our scheme, the users are still able to decode their data without knowing the channel gains or carrier offsets. We have also performed the theoretical SER analysis to predict the behavior of the system. Based on this analysis, a numerical power allocation is proposed to improve the performance of the cooperative communication system for the amplify-and-forward protocol.

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